Expert Systems With Applications 149 (2020) 113338

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Chimp optimization algorithm

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ARTICLE INFO

Article history:
Received 11 April 2019
Revised 11 January 2020
Accepted 24 February 2020
Available online 29 February 2020

Keywords:
Chimp
Mathematical model
Metaheuristic
Optimization

ABSTRACT

This paper proposes a novel metaheuristic algorithm called Chimp Optimization Algorithm (ChOA) inspired by the individual intelligence and sexual motivation of chimps in their group hunting, which is different from the other social predators. ChOA is designed to further alleviate the two problems of slow convergence speed and trapping in local optima in solving high-dimensional problems. In this paper, a mathematical model of diverse intelligence and sexual motivation of chimps is proposed. In this regard, four types of chimps entitled attacker, barrier, chaser, and driver are employed for simulating the diverse intelligence. Moreover, four main steps of hunting, i.e. driving, chasing, blocking, and attacking, are implemented. The proposed ChOA algorithm is evaluated in 3 main phases. First, a set of 30 mathematical benchmark functions is utilized to investigate various characteristics of ChOA. Secondly, ChOA was tested by 13 high-dimensional test problems. Finally, 10 real-world optimization problems were used to evaluate the performance of ChOA. The results are compared to several newly proposed meta-heuristic algorithms in terms of convergence speed, the probability of getting stuck in local minimums, and exploration, exploitation. Also, statistical tests were employed to investigate the significance of the results. The results indicate that the ChOA outperforms the other benchmark optimization algorithms.

1. Introduction

Metaheuristic Optimization Algorithms (MOAs) have become very popular in engineering applications. As the complicacy of problems increases, the need for new MOAs becomes obvious more than before. The reasons for this demand, can be summarized into five major motivations: (i) simple concepts and structure, which assists scientists to learn MOAs quickly and apply them to their problems; (ii) derivation-free mechanisms: this makes MOAs highly suitable for real-world engineering problems with costly or unknown gradient information; (iii) local optima avoidance: they have greater abilities to avoid local minima compared to conventional optimization algorithms; (iv) flexibility, which refers to the pertinence of MOAs to different problems without any specific changes in their structure (they assume problems as black boxes); (v) relatively simple and entirely effective hardware implementation, the majority of MOAs have parallel structures, therefore, hardware implementation and parallel computing (e.g. via Filed Programmable Gate Array (FPGA)) can strongly increase their performances.

Nature-inspired MOAs solve optimization problems by imitating physical or biological phenomena. They can be divided into four major categories: physics-based, evolution-based, swarm-based, and human-based methods. Evolutionary Algorithms (EAs) are usually inspired by the concepts of natural evolution. The search process initiates with a stochastically generated population which is evolved over following generations. The prominent feature of EAs is that the best individuals are always merged together to create the subsequent generation of individuals. This let the population be optimized over the course of generations. The most popular EA is Genetic Algorithms (GA) (Holland, 1992) that simulates the Darwinian evolution concepts. Some other popular EAs are Differential Evolution (DE) (Storn & Price, 1997), Evolution Strategy (ES) (Beyer & Schwefel, 2002), and Biogeography-Based Optimizer (BBO) (Khishe, Mosavi, & Kaveh, 2017; Kaveh et al., 2019).

Physics-based methods mimic the physical concepts in the world so that a stochastic set of search agents communicate and move entire search space pursuant physical concepts. Some of the most popular methods are Simulated Annealing (SA) (Kirkpatrick, Gelatt, & Vecchi, 1983), Big-Bang Big-Crunch (BBBC) (Erol & Eksin, 2006), Gravitational Search Algorithm (GSA) (Rashedi, Nezamabadi-Pour, & Saryazdi, 2009), Chaotic Fractal Walk Trainer (Khishe, Mosavi, & Moridi, 2018), and Adaptive Best-mass Gravitational Search Algorithm (Mosavi, Khishe, Parvizi, Naseri, & Ayat, 2019).

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https://doi.org/10.1016/j.eswa.2020.113338
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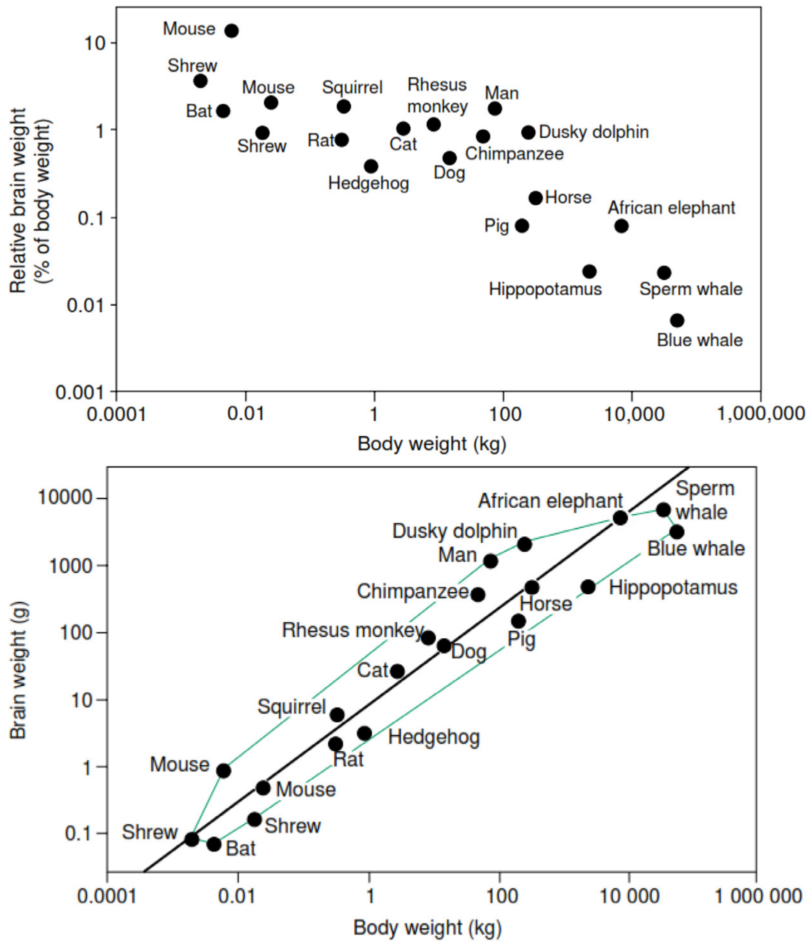


Fig. 1. Two different plot of relationship between body size and brain size in various mammals.

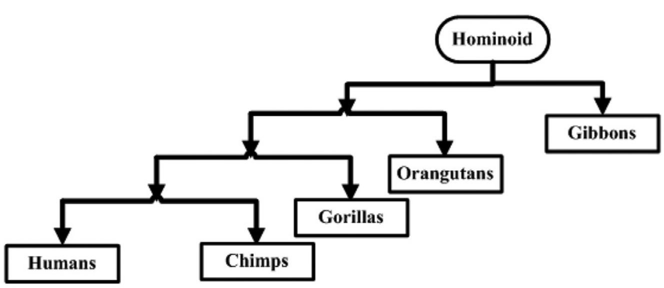


Fig. 2. Phylogeny of super-family Hominoid.

The third group of MOAs includes algorithms inspired by human behaviours in the literature. Some of the most popular techniques are Tabu (Taboo) Search (TS) (Osman, 1993), Imperialist Competitive Algorithm (ICA) (Atashpaz-Gargari & Lucas, 2007), Teaching Learning Based Optimization (TLBO) (Rao, Savsani, & Vakharia, 2011), Interior Search Algorithm (ISA) (Ravakhah, Khishe, Aghababaee, & Hashemzadeh, 2017), Innovative Gunner (AIG) (Pijarski & Kacejko, 2019).

The forth group of MOAs includes Swarm Intelligence-based Algorithms (SIAs) that originates from natural behaviour of animals in their herds, flock, colonies, and schools. The most popular algorithm, in this category, is Particle Swarm Optimization (PSO) (Han, Lu, Hou, & Qiao, 2016). Two other popular swarm-based algorithms are Ant Colony Optimization (ACO) (Dorigo, Birattari, & Stutzle, 2006) and Artificial Bee Colony (ABC) (Basturk & Karaboga, 2006).

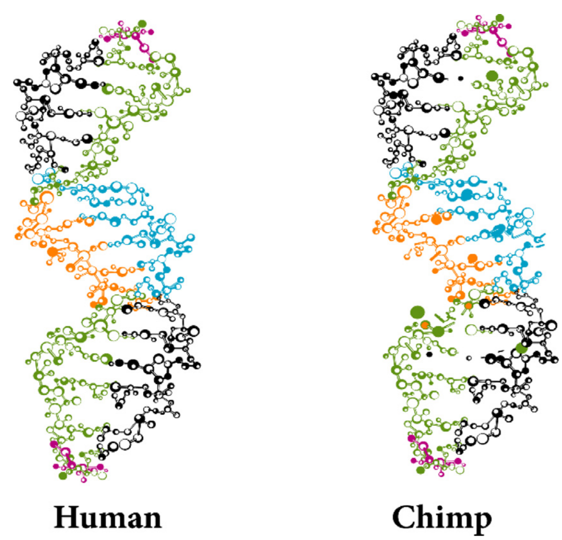


Fig. 3. Human and chimp DNA.

Other recently proposed SIAs are Cuckoo Search (CS) (Yang & Deb, 2009), Bat-inspired Algorithm (BA) (Yang, 2010), Firefly Algorithm (FA) (Yang, 2010), Krill Herd (KH) (Gandomi & Alavi, 2012), Grey Wolf Optimizer (GWO) (Emary, Zawbaa, & Grosan, 2017), GWO with LevyŕFlight (Heidari & Pahlavani, 2017), chaotic GWO (Heidari & Abbaspour, 2017), evolutionary population dynamics

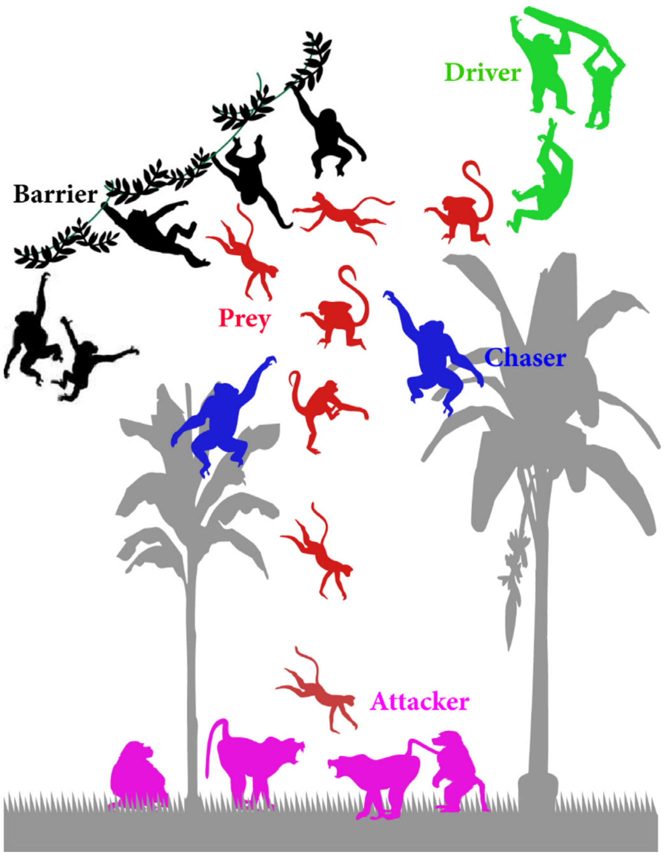


Fig. 4. The first phase of hunting process (exploration).

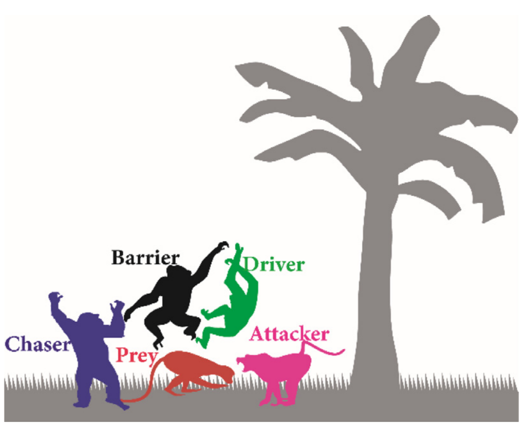
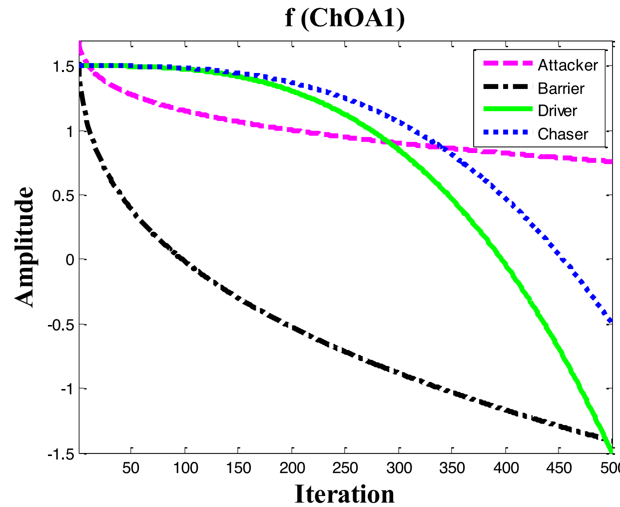
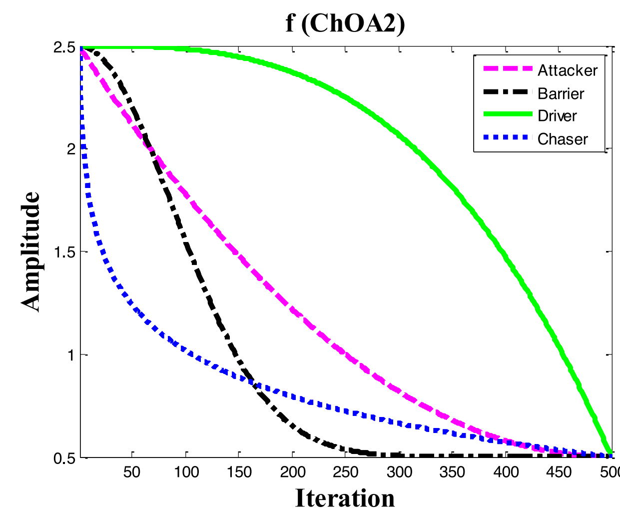


Fig. 5. The second phase of hunting process (exploitation).



(a) ChOA1



(b) ChOA2

Fig. 6. Mathematical models of dynamic coefficients (f) related to independent groups for (a) ChOA1 and (b) ChOA2.

Table 1
The dynamic coefficient of **f** vector.

Groups	ChOA1	ChOA2
Group1	$1.95-2t^{1/4}/T^{1/3}$	$2.5-(2\log(t)/\log(T))$
Group2	$1.95-2t^{1/3}/T^{1/4}$	$(-2t^3/T^3)+2.5$
Group3	$(-3t^3/T^3)+1.5$	$0.5+2\exp[-(4t/T)^2]$
Group4	$(-2t^3/T^3)+1.5$	$2.5+2(t/T)^2-2(2t/T)$

and Grasshopper Optimization Approaches (GOA) (Mafarja et al., 2017), binary Salp Swarm Algorithm (BSSA) with crossover scheme (Farisa et al., 2018), hybrid GOA and MLP (Heidari, Farisa, Aljarah, & Mirjalili, 2019), hybrid binary Ant Lion Optimizer (ALO) with rough set and approximate entropy (Mafarja & Mirjalili, 2019), hybrid MLP and Salp Swarm Algorithm (MLP-SSA) (Khishe & Mohammadi, 2019), improved Monarch Butterfly Optimization (MBO) algorithm (Sun, Chen, Xu, & Tian, 2019), Improved Whale Trainer (IWT) (Khishe & Mosavi, 2019), and hybrid Dragonfly Optimization Algorithm and MLP (DOA-MLP) (Khishe & Saffari, 2019).

This category of MOAs started to be interesting since PSO was proven to be very competitive with EAs, human-based, and physical-based methods. Totally, SIAs have some advantages over other MOAs that are listed below:

- SIAs memorize search space information over the course of iteration while EAs discard any information of the prior generations.
- SIAs almost use memory to keep the best solution acquired so far.
- SIAs generally have fewer parameters to adjust compare to other MOAs.
- SIAs have fewer operators compared to EAs (crossover, mutation, immigration, and so on).
- SIAs are easier to implement than the other MOA groups.

In spite of the demand for more function evaluation, the literature shows that SIAs are highly appropriate for solving real

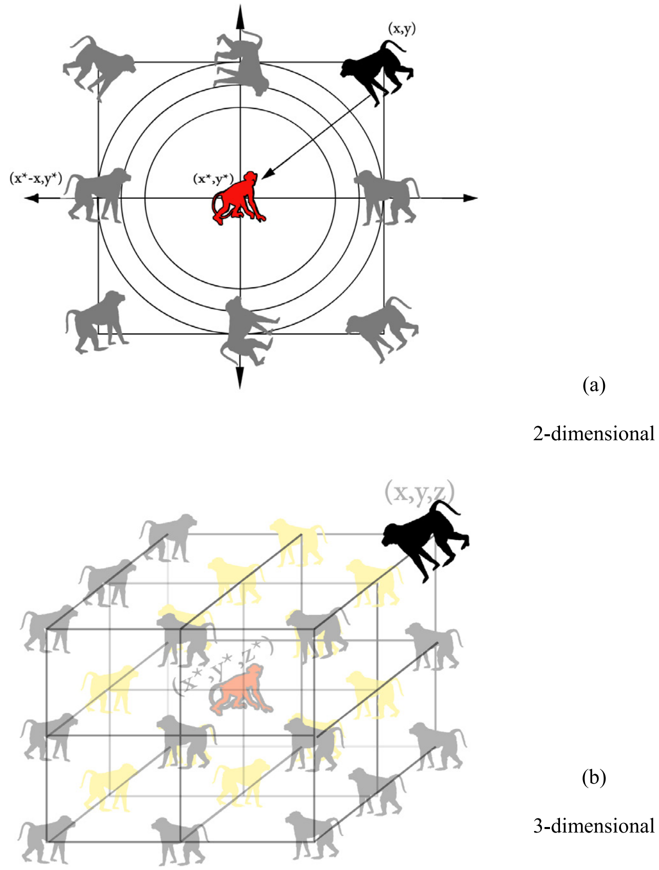


Fig. 7. Two and three-dimensional position vectors and their possible next locations.

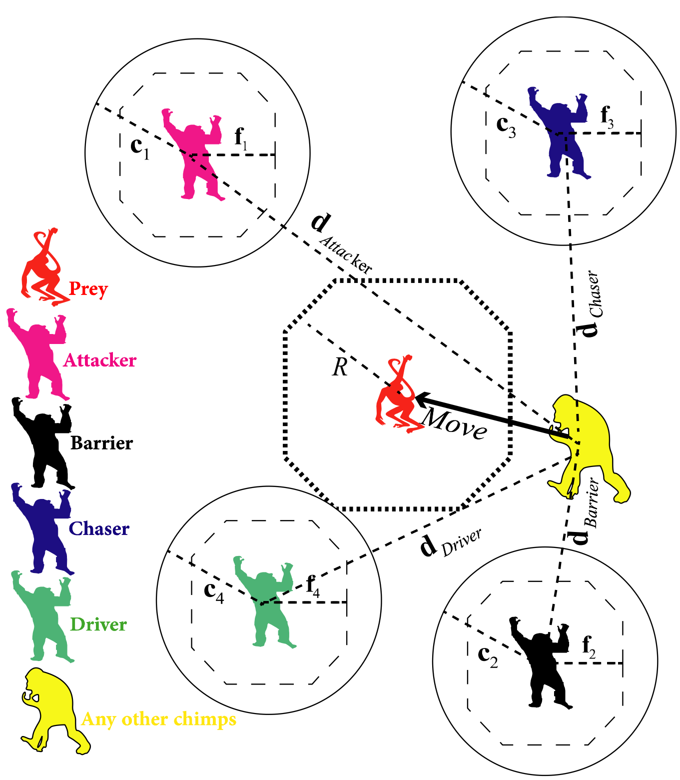


Fig. 8. Position updating in ChOA.

world engineering problems since they are able to elude local minima, exploring the search space more complete, and exploiting the global optimum more reliable than any other MOAs. In addition, the No Free Lunch (NFL) theorem shows that all the MOAs execute equally on all optimization problems (Wolpert & Macready, 1997). Hence, there are still problems that have not been solved, or they can be resolved better by new MOAs. The main motivations of this article are these two reasons, in which a novel SIAs is proposed and compared to the current well-known MOAs in the literature.

In spite of the considerable number of recently proposed publications in this field, there are yet other intelligent swarming behaviours in nature that have not obtained merit attention. One of the amazing swarming behaviours in nature is the Intelligent Group Haunting (IGH) of chimps. Since there is no research in the literature to simulate the IGH of chimps, this article aims to first discover the main characteristics of chimps' IGH. An MOA is then proposed based on the modelled IGH called Chimp Optimization Algorithm (ChOA). In addition to NFL theorem underpinning work motivation, the main reasons for choosing chimps from among numerous swarming behaviour are individual intelligence and sexual motivation. These two vary from the other hunters in nature.

Irrespective of the differences between the MOAs, a common characteristic is the division of the search producer into two phases: exploration and exploitation. The exploration phase refers to the producer of investigating the search space as widely as possible. A MOA needs to have random operators to stochastically and globally discover the search space in order to reinforce this phase. Nevertheless, exploitation refers to the local search ability around the promising areas gained in the exploration phase. Having strong operators within these two phases or finding a proper balance between them is considered a challenging point in the literature (Mirjalili, 2015, 2016).

The main differences between the social behaviour of chimps and any other flocking behaviours are:

1) Individuals' Diversity: In a group of chimps, individuals are not basically quite similar in terms of ability and intelligence, but they all perform their tasks as members of a hunting group. Each individual's ability can be useful in a special phase of the hunting event. Therefore, a chimp according to his special ability takes responsibility for a part of hunt (Stanford, 1996). In this article, a mathematical model of diverse chimps called independent chimps is proposed. In other words, various models with diverse curvatures, slopes, and interception points are utilized to give chimps different behaviours as in natural hunting duties. Independent chimps can improve the exploration phase by discovering the searching space more thoroughly.
2) Sexual Motivation: As well as nutritional advantages of group hunting, it has also been proved that chimps' hunting is affected by the probable social benefits of obtaining meat (Stanford et al., 1994). Acquiring meat provides an opportunity to trade it in return for social favours, e.g. sex and grooming. This incentive in the final stage causes chimps to forget their responsibilities in hunting process. Therefore, they try to obtain meat chaotically. This unconditional behaviour in final stage lead to improve exploitation phase and convergence rate.

To sum up, the main contribution of the paper can be categorized as follow:

- **Stage 1:** According to the comprehensive background study in literature, MOAs are categorized into four main groups as physics-based, evolution-based, swarm-based, and human-based. The result of this stage was choosing a swarm-based algorithm based on their ability and our target.
- **Stage 2:** A comprehensive study has been done to choose a special creature that has not been previously modelled and also

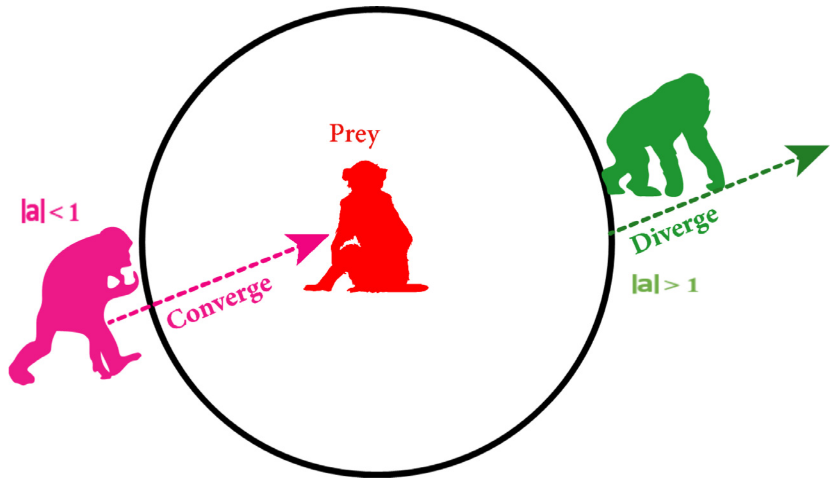


Fig. 9. Position updating mechanism of chimps and effects of |a| on it.

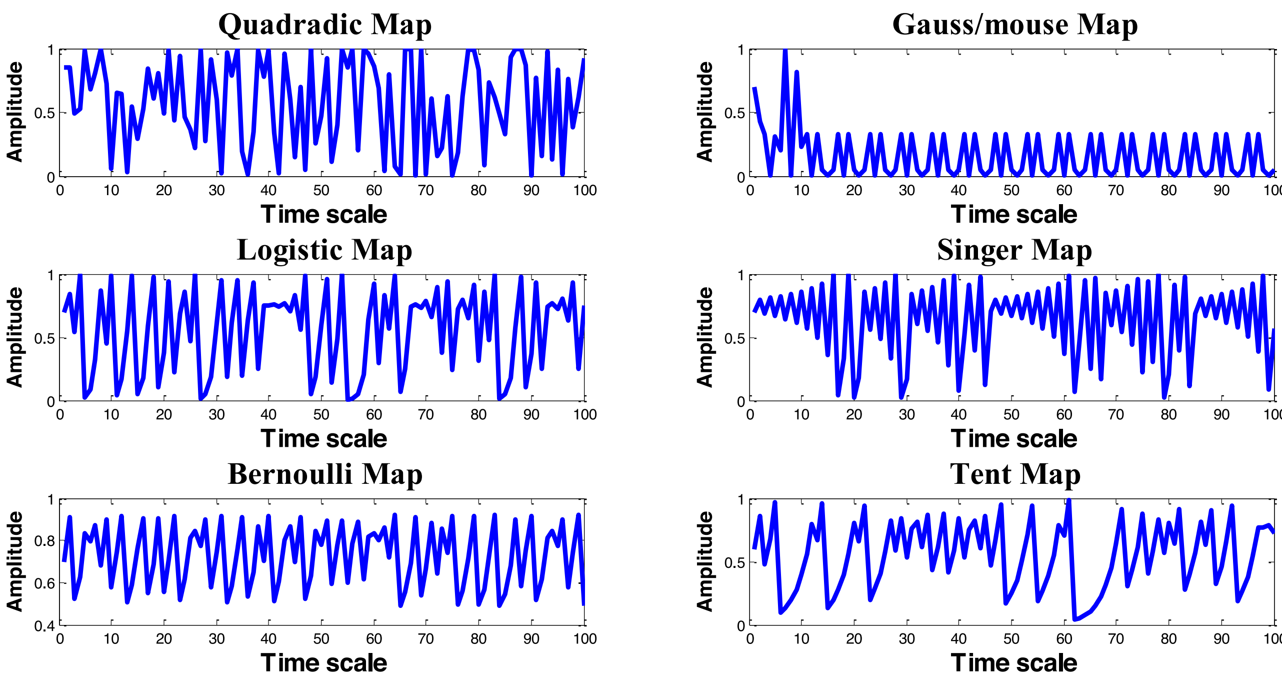


Fig. 10. The chaotic maps used in the article.

Table 2
Chaotic maps.

No	Name	Chaotic map	Range
1	Quadratic	$x_{i+1} = {x^2}_i - c, \ c = 1$	(0,1)
2	Gauss/mouse	$x_{i+1} = \begin{cases} 1 & x_i = 0 \\ \frac{1}{\mathrm{mod}\,(x_i, 1)} & \text{otherwise} \end{cases}$	(0,1)
3	Logistic	$x_{i+1} = \alpha x_i(1 - x_i), \alpha = 4$	(0,1)
4	Singer	$x_{i+1} = \mu(7.86x_i - 23.31x_i^2 + 28.75x_i^3 - 13.302875x_i^4), \ \mu = 1.07$	(0,1)
5	Bernoulli	$x_{i+1} = 2x_i(mod 1)$	(0,1)
6	Tent	$x_{i+1} = \begin{cases} \frac{x_i}{0.7} & x_i < 0.7 \\ \frac{10}{3}(1 - x_i) & 0.7 \le x_i \end{cases}$	(0,1)

having special intelligent behaviour. So, the result of this stage was choosing chimp and it's Intelligent Group Haunting (IGH).

- **Stage 3:** Discovering and modelling the main characteristics of Intelligent Group Haunting (IGH) of chimps (i.e.: diverse intelligence and sexual motivation)
- **Stage 4:** The implementation of four main steps of hunting as (driving, chasing, blocking, and attacking)
- **Stage 5:** Evaluation of the proposed ChOA algorithm by 30 mathematical benchmark functions, 13 high-dimensional test problems, and 10 real-world optimization problems.

The rest of the paper is structured as follows. Section 2 describes the chimp optimization algorithm developed in the article. Optimization problems and their experimental results are pre-

Table 3
Unimodal benchmark function.

Function	Dim	Range	f_{min}
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30, 100	[− 100, 100]	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30, 100	[− 10, 10]	0
$F_3(x) = \sum_{i=1}^{n} (\sum_{j-1}^{i} x_j)^2$	30, 100	[− 100, 100]	0
$F_4(x) = \max_i \{\lvert x_i \rvert, 1 \le i \le n\}$	30, 100	[− 100, 100]	0
$F_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30, 100	[− 30, 30]	0
$F_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	30, 100	[− 100, 100]	0
$F_7(x) = \sum_{i=1}^{n} i x_i^4 + random[0, 1)$	30, 100	[− 1.28, 1.28]	0

Algorithm: ChOA

Initialize the chimp population x_i (i=1,2, ...,n)
Initialize **f, m, a** *and* **c**
Calculate the position of each chimp
Divide chimps randomly into independent groups
***Until** stopping condition is satisfied*
Calculate the fitness of each chimp
$\mathbf{x}_{Attacker}$=*the best search agent*
$\mathbf{x}_{Chaser}$=*the second best search agent*
$\mathbf{x}_{Barrier}$=*the third best search agent*
$\mathbf{x}_{Driver}$=*the fourth best search agent*
***while** (t < maximum number of iterations)*
***for** each chimp:*
Extract the chimp's group
Use its group strategy to update **f, m** *and* **c**
Use **f, m** *and* **c** to calculate **a** *and then* **d**
end for
***for** each search chimp*
***if** (μ<0.5)*
***if** (|**a**| < 1)*
Update the position of the current search agent by the Eq. (2)
***else if** (|**a**|>1)*
Select a random search agent
end if
***else if** (μ>0.5)*
Update the position of the current search by the Eq.(9)
end if
end for
Update **f, m, a** *and* **c**
Update $\mathbf{x}_{Attacker}, \mathbf{x}_{Driver}, \mathbf{x}_{Barrier}, \mathbf{x}_{Chaser}$
t=t+1
end while
return $\mathbf{x}_{Attacker}$

Fig. 11. Presents the pseudo-code of ChOA.

sented and discussed in Sections 3. Finally, Section 4 concludes the work and suggests directions for further research.

2. Chimp optimization algorithm

This section presents and discusses the inspiration of ChOA method. Afterwards, it provides the mathematical model of the proposed algorithm.

2.1. Inspiration

Chimps (sometimes called Chimpanzees) are one of two merely African species of great ape. They are as much as the closest to the humans' living relatives. As shown in Fig. 1, the chimps, as well as the dolphins, have the most similar Brain to Body Ratio (BBR) to humans. As discussed in Roth and Dicke (2005) mammals with relatively larger BBR are mostly assumed to be smarter. The chimp and the human DNA are so similar because they are descended from a single ancestor species (Hominoid) that lived seven or eight million years ago. Fig. 2 indicates the phylogeny of superfamily Hominoid (Israfil, Zehr, Mootnick, Ruvolo, & Steiper, 2011). As shown in Fig. 3, these two species share a 98.8% of their DNAs (Tomkins & Bergman, 2012).

The chimp's colony is a fission-fusion society. This kind of society is one in which the combination or size of the colony changes as time passes and members move throughout the environment. For chimps that live in fission-fusion colonies, group composition is a dynamic property (Couzin & Laidre, 2009). Considering these issues, the independent group concept is proposed. In this technique, each group of the chimps independently attempts to discover the search space with its own strategy. In each group, chimps are not quite similar in terms of ability and intelligence, but they are all doing their duties as a member of the colony. The ability of each individual can be useful in a particular situation.

In a chimp colony, there are four types of chimps entitled driver, barrier, chaser, and attackers. They all have different abilities, but these diversities are necessary for a successful hunt. Drivers follow the prey without attempting to catch up with it. Barriers place themselves in a tree to build a dam across the progression of the prey. Chasers move rapidly after the prey to catch up with it. Finally, attackers prognosticate the breakout route of the prey to infliction it (the prey) back towards the chasers or down into the lower canopy. These steps of hunting process are shown in Fig. 4. Attackers are thought to need much more cognitive endeavour in prognosticating the subsequent movements of the prey, and they are thus remunerated with a larger piece of meat after a successful hunt. This important role (attacking) correlates positively with the age, smartness, and physical ability. Moreover, chimps can change duties during the same hunt or keep their same duty during the entire process (Boesch, 2002).

It has been proven that chimps hunt to obtain meat for trading in social favours such as coalitionary support, sex or grooming (Stanford et al., 1996). So, by opening up a new realm of privileges, smartness may have an indirect effect on hunting. To the best of our knowledge, in addition to humans, this social incentives has been proposed only for chimps. Hence, it would represent a critical difference between chimps and other social predators that depend on cognitive ability. This social incentive (sexual motivation) causes the chimps to act chaotically in the final stage of hunting process so that all chimps abandon their special duties and they try to get meat, frantically. Generally speaking, the hunting process of chimps is divided into two main phases: Exploration which consists of driving, blocking and chasing the prey and Exploitation which consists of attacking the prey. These two phases are shown in Figs. 4 and 5, respectively. Then, all of these concepts of ChOA are mathematically formulated in the following section.

2.2. Mathematical model and algorithm

In this section, mathematical models of independent group, driving, blocking, chasing and attacking are presented. Corresponding ChOA algorithm is then specified.

2.2.1. Driving and chasing the prey

As mentioned above, the prey is hunted during the exploration and exploitation phases. To mathematically model driving and chasing the prey, Eqs. (1) and (2) are proposed.

$$\mathbf{d} = \left| \mathbf{c}.\mathbf{x}_{\text{prey}}(t) - \mathbf{m}.\mathbf{x}_{chimp}(t) \right| \quad (1)$$

$$\mathbf{x}_{chimp}(t+1) = \mathbf{x}_{\text{prey}}(t) - \mathbf{a}.\mathbf{d} \quad (2)$$

Where t indicates the number of current iteration, **a, m**, and **c** are the coefficient vectors, $\mathbf{X_{prey}}$ is the vector of prey position and $\mathbf{X_{chimp}}$ is the position vector of a chimp. **a, m**, and **c** vectors are calculated by the Eqs. (3)–(5), respectively.

$$\mathbf{a} = 2.\mathbf{f}.\mathbf{r}_1 - \mathbf{f} \quad (3)$$

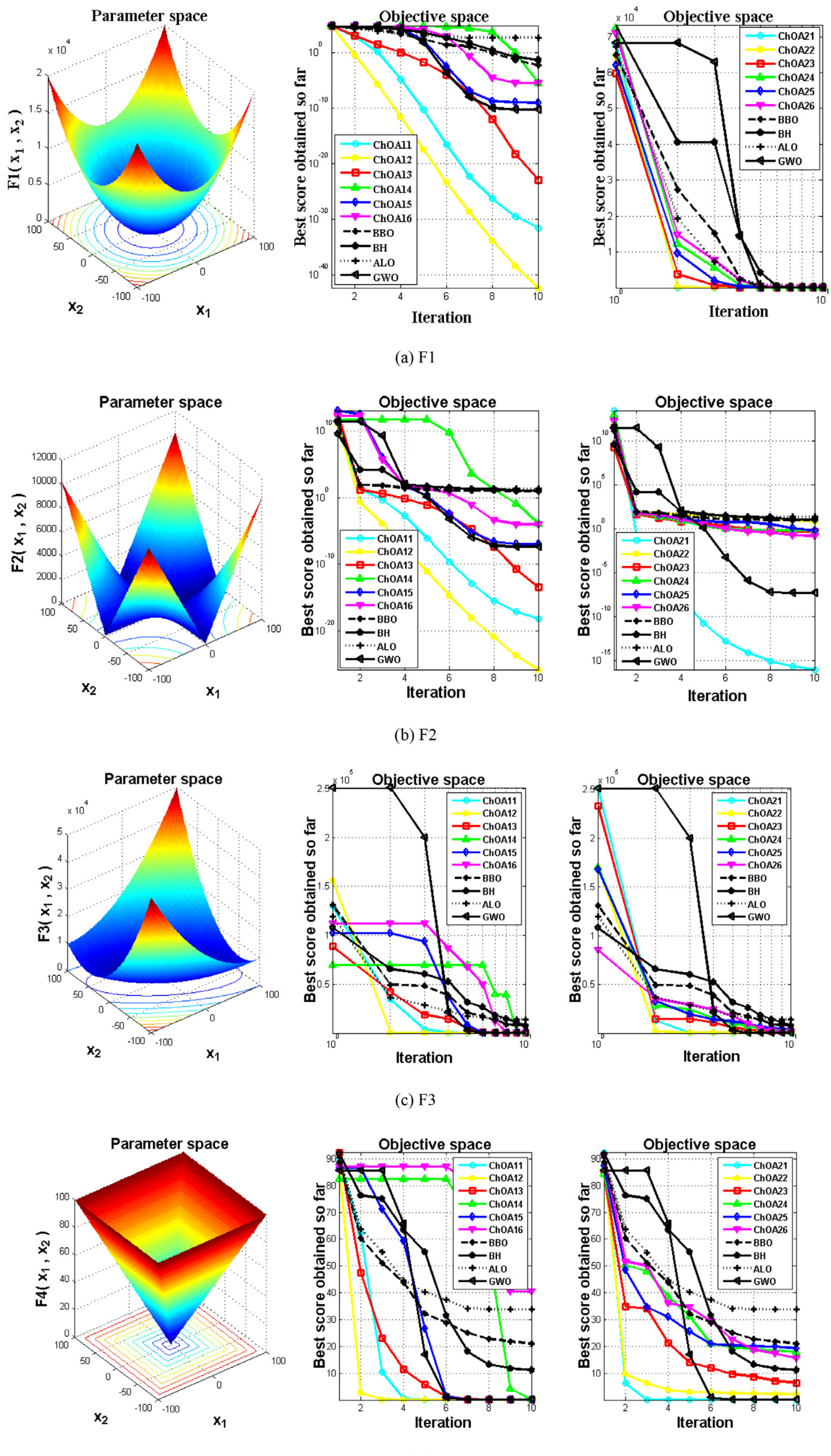
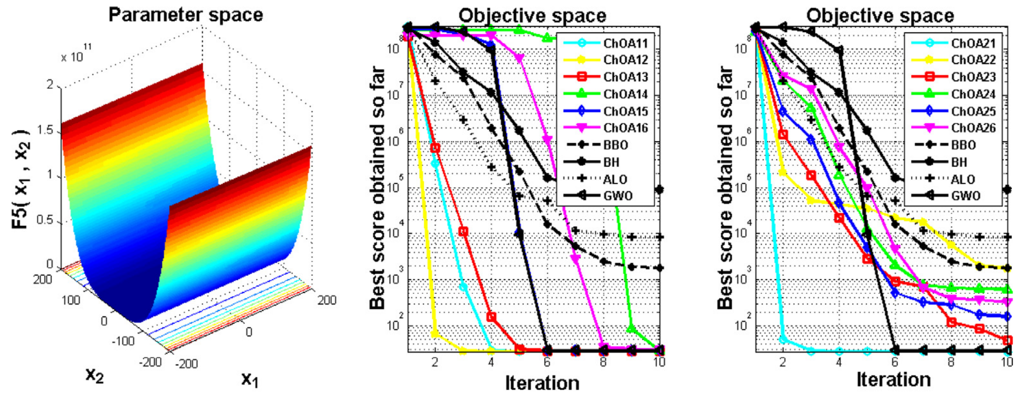
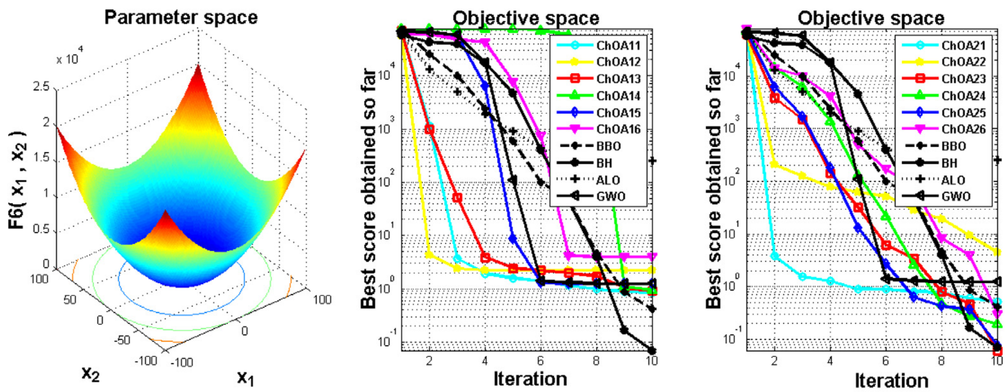


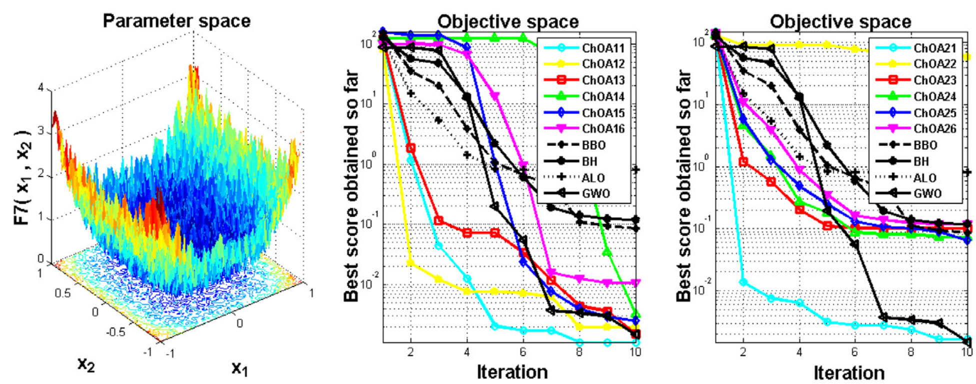
Fig. 12. Convergence curve of algorithms on the unimodal test functions.



(e) F5



(f) F6



(g) F7

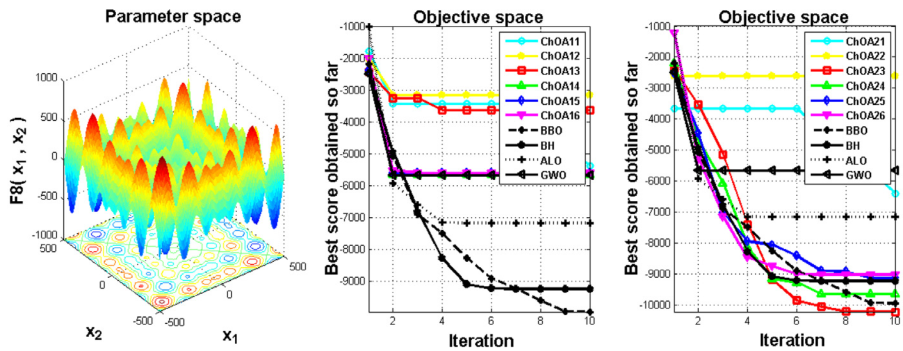
Fig. 12. Continued

$$\mathbf{c} = 2.\mathbf{r}_2 \tag{4}$$

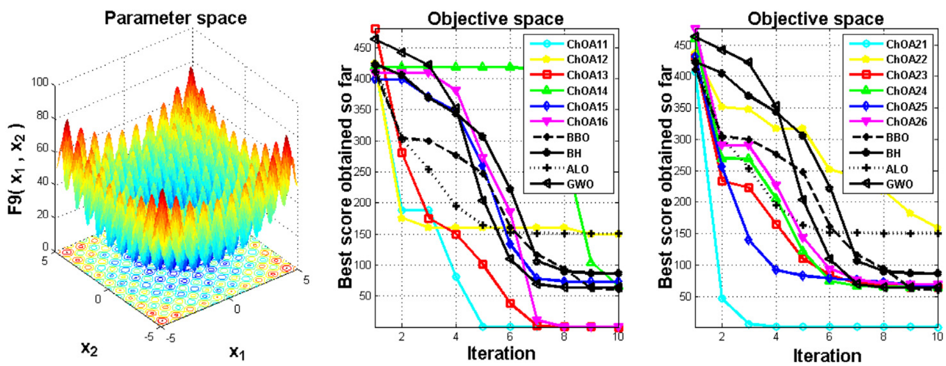
$$\mathbf{m} = \mathbf{Chaotic\_value} \tag{5}$$

In which, **f** is reduced non-linearly from 2.5 to 0 through the iteration process (in both exploitation and exploration phase). Where $\mathbf{r_1}$ and $\mathbf{r_2}$ are the random vectors in the range of [0,1]. Finally, **m** is a chaotic vector calculated based on various chaotic map so that this vector represents the effect of the sexual motivation of chimps in the hunting process. A full description of this vector will be described in detail in the following subsections. In the conventional population-based optimization algorithm, all particles have similar behaviour in local and global searches so that the individuals can be considered as a single group with one common search strategy. However, theoretically, in every population-based optimization algorithm, different independent groups that have a common goal can be used to have a direct and random search result at the same time. In the following, independent groups of chimp using different strategies to update **f** will be modelled mathematically. Updating the independent groups can be implemented by any continuous function. These functions must be chosen in such a way that during each iteration **f** is reduced (Mirjalili, Lewis, & Sadiq, 2014).

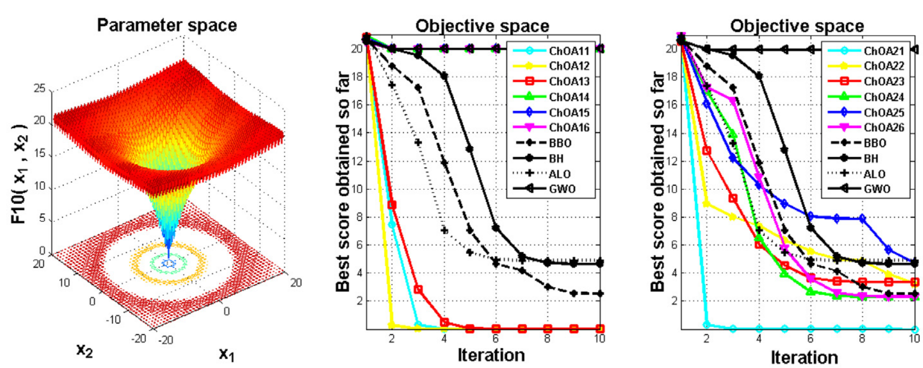
These four independent groups use their own patterns to search the problem space locally and globally. Also among various strategies which have been tested, two different versions of ChOA with



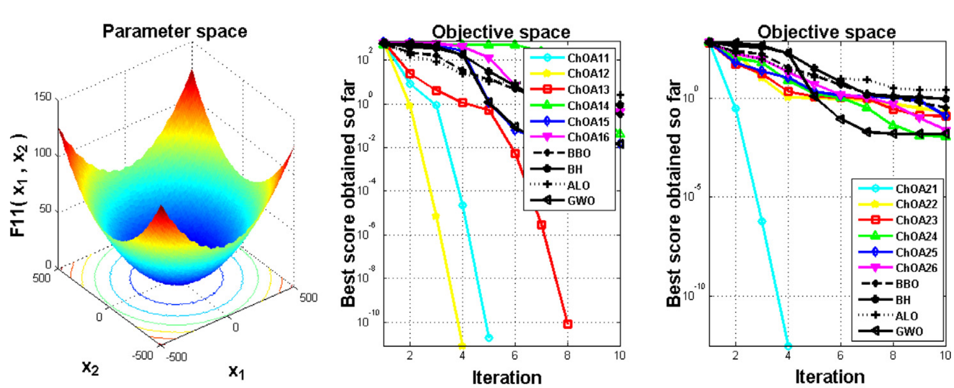
(a) F_8



(b) F_9

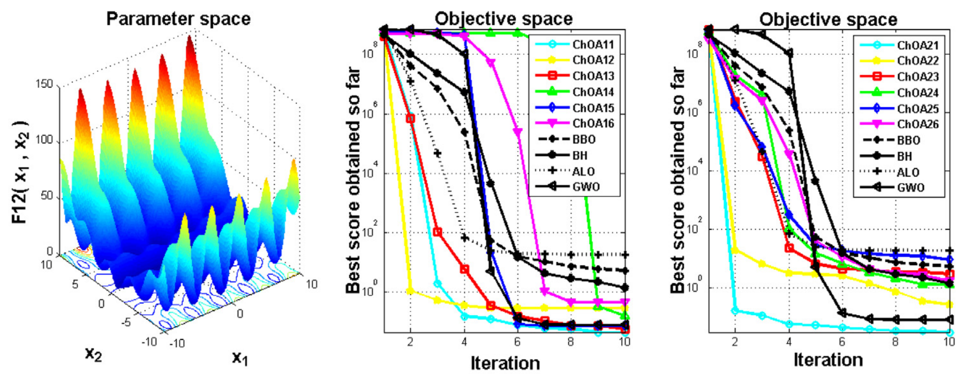


(c) F_{10}

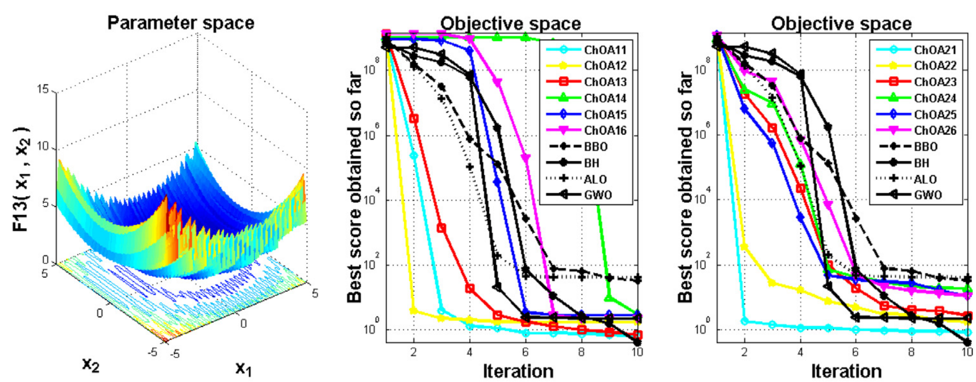


(d) F_{11}

Fig. 13. Convergence curve of algorithms on the multimodal test functions.



(e) F_{12}



(f) F_{13}

Fig. 13. Continued

various independent groups called ChOA1 and ChOA2 are selected to have the best performance in the benchmark optimization problems. The dynamic coefficients of **f** have been proposed in Table 1 and Fig. 6. In this table, T represents the maximum number of iterations, and t indicates the current iteration. These dynamic coefficients have been chosen with various curves and slopes so that each independent group has specific searching behaviour for the sake of improving the performance of ChOA.

Some points may be considered to understand how independent groups are effective in ChOA:

- Independent groups have different strategies to update **f**, so chimps could explore the search space with different capability.
- Diverse and dynamic strategies of **f** cause balancing between global and local search.
- Independent groups contain non-linear strategies such as logarithmic and exponential functions for **f**, so ChOA could be effective in solving complex optimization problems.
- ChOA with independent groups could be adaptable in solving a wider range of optimization problems.

To understand the effects of Eqs. (1) and (2), a two-dimensional representation of the position vector and a number of possible neighbours are shown in Fig. 7a. As can be observed, a chimp in position (x,y) can change its position with respect to prey's (x^*, y^*)location. Various locations around the most suitable agent can be taken considering its current location and changing and setting the values of **a** and **c** vectors. For instance the location of $(x^* - x, y^*)$ is obtained by setting $\boldsymbol{a} = (1,0)$, $\boldsymbol{m} = (1,1)$ and $\boldsymbol{c} = (1,1)$. Updated possible locations of a chimp in a three-dimensional space are indicated in Fig. 7b. It should be noted that the chimps are allowed to access any position between the points shown in Fig. 7 through the random vectors $\mathbf{r}_1$ and $\mathbf{r}_2$. So, any chimp can randomly change its location within the space surrounding the prey using Eqs. (1) and (2).

This concept can be generalized to an n-dimensional search space. As mentioned in the previous section, the chimps also attack the prey with the chaotic strategy. This method is mathematically formulated in the following section.

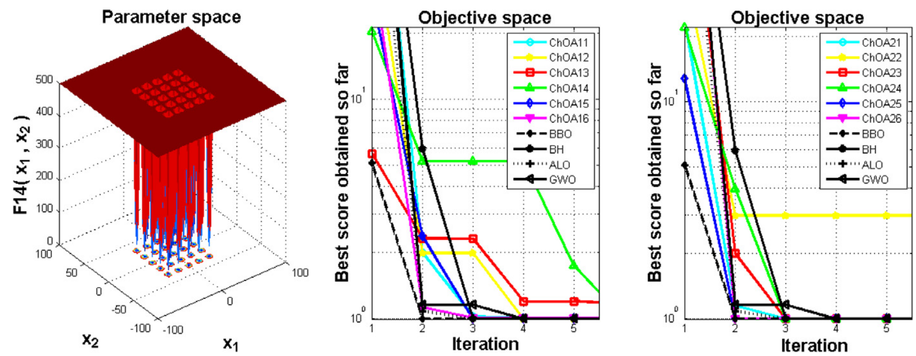
2.2.2. Attacking method (exploitation phase)

To mathematically model attacking behaviour of chimps, two approaches are designed as follows: The chimps are capable of exploring the prey's location (by driving, blocking and chasing) and then encircling it. The hunting process is usually conducted by attacker chimps. Driver, barrier and chaser chimps are occasionally participate in the hunting process. Unfortunately in an abstract search space there is no information about the optimum location (prey). In order to mathematically simulate the behaviour of the chimps, it is assumed that the first attacker (best solution available), driver, barrier and chaser are better informed about the location of potential prey. So, four of the best solutions yet obtained is stored and other chimps are forced to update their positions according to the best chimps locations. This relationship is expressed by the Eqs. (6)–(8).

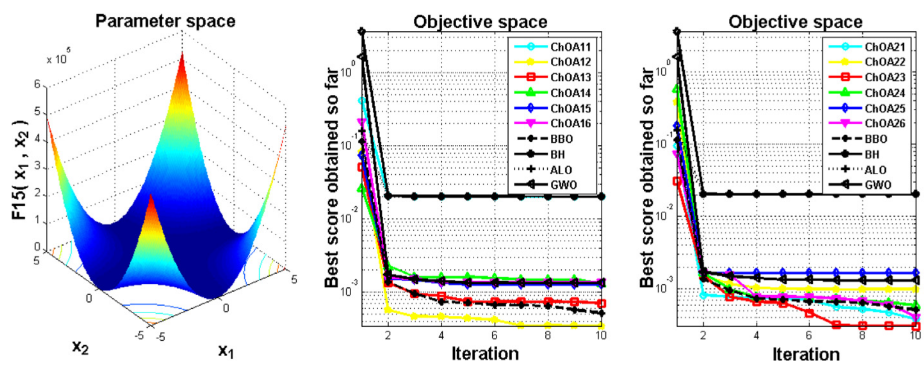
$$\mathbf{d}_{Attacker} = |\mathbf{c}_1\mathbf{x}_{Attacker} - \mathbf{m}_1\mathbf{x}|, \mathbf{d}_{Barrier} = |\mathbf{c}_2\mathbf{x}_{Barrier} - \mathbf{m}_2\mathbf{x}|,$$
$$\mathbf{d}_{Chaser} = |\mathbf{c}_3\mathbf{x}_{Chaser} - \mathbf{m}_3\mathbf{x}|, \mathbf{d}_{Driver} = |\mathbf{c}_4\mathbf{x}_{Driver} - \mathbf{m}_4\mathbf{x}|. \quad (6)$$

$$\mathbf{x}_1 = \mathbf{x}_{Attacker} - \mathbf{a}_1(\mathbf{d}_{Attacker}), \mathbf{x}_2 = \mathbf{x}_{Barrier} - \mathbf{a}_2(\mathbf{d}_{Barrier}),$$
$$\mathbf{x}_3 = \mathbf{x}_{Chaser} - \mathbf{a}_3(\mathbf{d}_{Chaser}), \mathbf{x}_4 = \mathbf{x}_{Driver} - \mathbf{a}_4(\mathbf{d}_{Driver}). \quad (7)$$

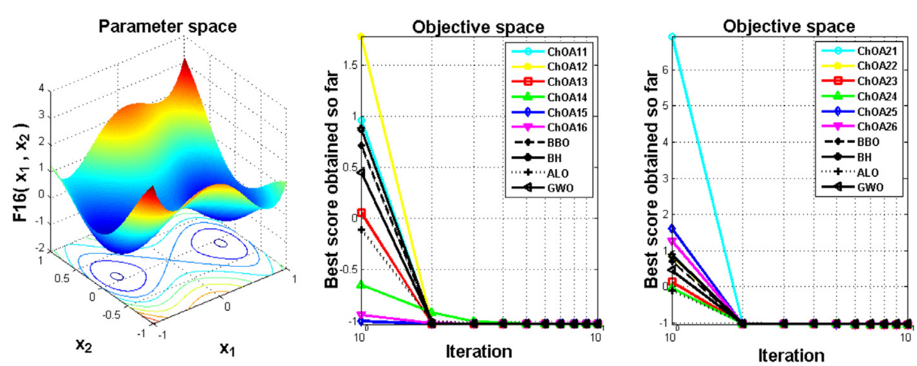
$$\mathbf{x}(t+1) = \frac{\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 + \mathbf{x}_4}{4} \quad (8)$$



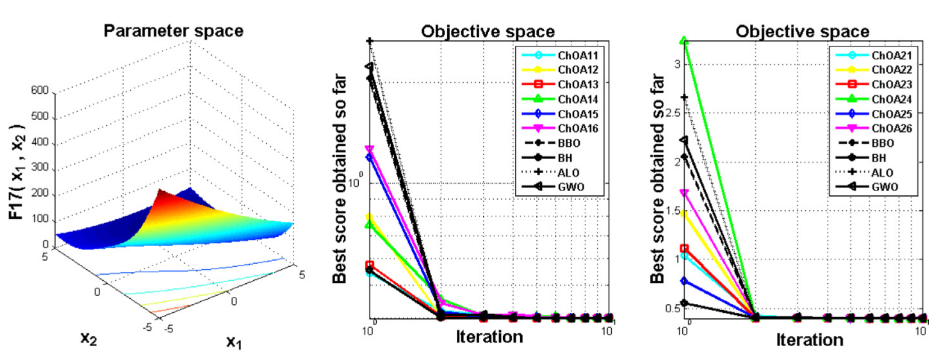
(a) F_{14}



(b) F_{15}

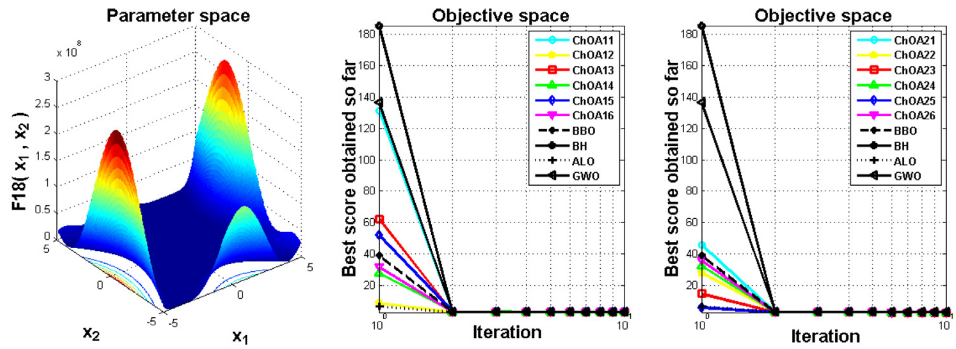


(c) F_{16}

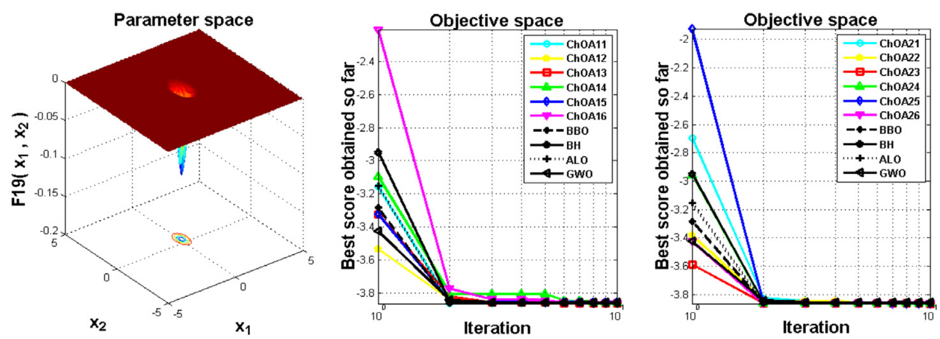


(d) F_{17}

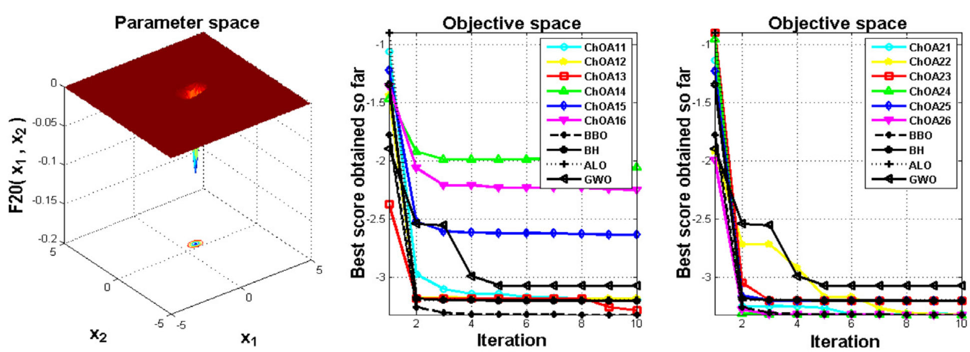
Fig. 14. Convergence curve of algorithms on the fixed-dimension multimodal benchmark functions.



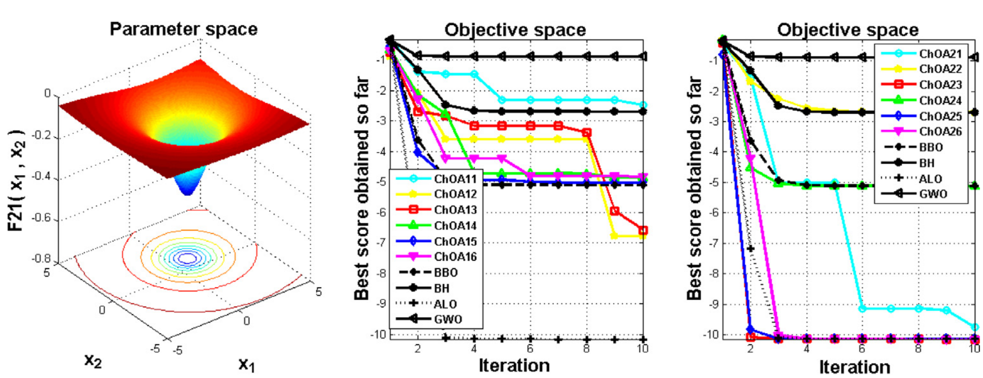
(e) F_{18}



(f) F_{19}

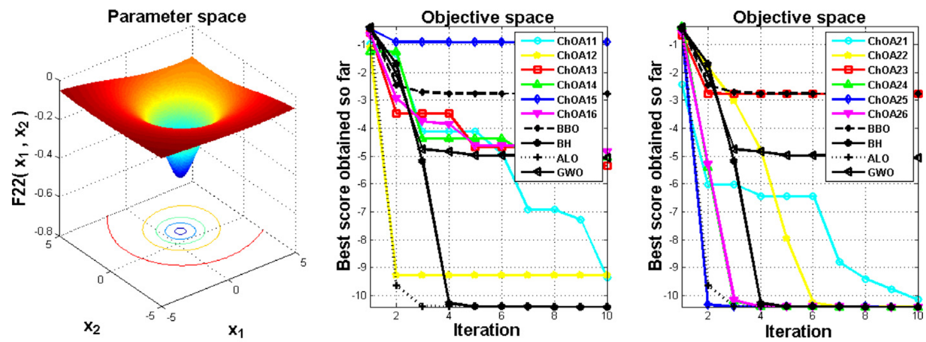


(g) F_{20}

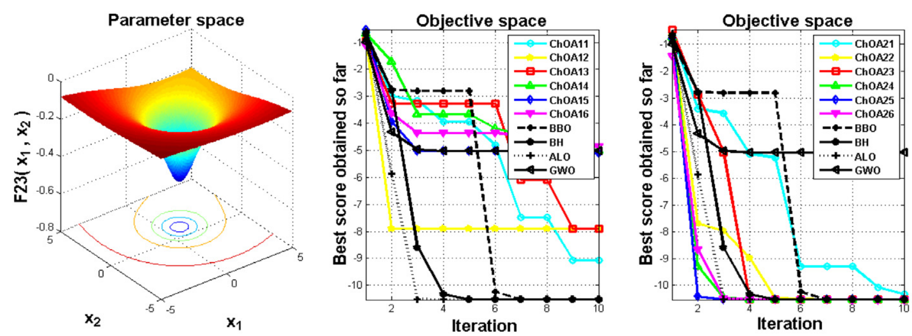


(h) F_{21}

Fig. 14. Continued



(i) F_{22}



(j) F_{23}

Fig. 14. Continued

Table 4
Multimodal benchmark function.

Function	Range	Dim	f_{min}
$F_8(x)=\sum_{i=1}^{n}-x_i\sin(\sqrt{\lvert x_i\rvert})$	[− 500, 500]	30,100	-418.9829 × Dim
$F_9(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	[− 5.12, 5.12]	30,100	0
$F_{10}(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}) - \exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+e$	[− 32, 32]	30,100	0
$F_{11}(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$	[− 600, 600]	30,100	0
$F_{12}(x)=\frac{\pi}{n}\{10\sin(\pi y_1)+\sum_{i=1}^{n-1}\begin{matrix}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]^2\\+(y_n-1)\end{matrix}\}+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4}$ $u(x_i,a,k,m)=\begin{cases}k(x_i-a)^m & x_i>a\\0 & -a<x_i<a\\k(-x_i-a)^m & x_i<-a\end{cases}$	[− 50, 50]	30,100	0
$F_{13}(x)=0.1\{\sin^2(3\pi x_1)+\sum_{i=1}^{n}\begin{matrix}(x_i-1)^2[1+\sin^2(3\pi x_i+1)]\\+(x_n-1)^2[1+\sin^2(2\pi x_n)]\end{matrix}\}+\sum_{i=1}^{n}u(x_i,5,100,4)$	[− 50, 50]	30,100	0

Table 5
Fixed-dimension multimodal benchmark function.

Function	Range	Dim	f_{min}
$F_{14}(x)=(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6})^{-1}$	[− 65, 65]	2	1
$F_{15}(x)=\sum_{i=1}^{11}[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}]^2$	[− 5, 5]	4	0.00030
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	[− 5, 5]	2	− 1.0316
$F_{17}(x)=(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6)^2+10(1-\frac{1}{8\pi})\cos x_1+10$	[− 5, 5]	2	0.398
$F_{18}(x)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)] \times[30+(2x_1-3x_2)^2\times(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	[− 2, 2]	2	3
$F_{19}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2)$	[1, 3]	3	− 3.86
$F_{20}(x)=-\sum_{i=1}^{4}c_i\exp(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2)$	[0, 1]	6	− 3.32
$F_{21}(x)=-\sum_{i=1}^{5}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	[0, 10]	4	− 10.1532
$F_{22}(x)=-\sum_{i=1}^{7}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	[0, 10]	4	− 10.4028
$F_{23}(x)=-\sum_{i=1}^{10}[(X-a_i)(X-a_i)^T+c_i]^{-1}$	[0, 10]	4	− 10.5363

Table 6
The rotated and shifted benchmark functions.

Function	Range	Dim	f_{min}
$F_{24}(x) = \sin(x_1)e^{[(1-cos(x_2))^2]} + \cos(x_2)e^{[(1-\sin(x_1))^2]} + (x_1 - x_2)^2$	$[-2\pi, 2\pi]$	4	-106.764537
$F_{25}(x) = 0.5 + \frac{\sin^2[\sqrt{{x_1}^2+{x_2}^2}]-0.5}{[1+0.001({x_1}^2+{x_2}^2)]^2}$	$[-100, 100]$	40	0.5
$F_{26}(x) = \sum_{i=1}^{m-1} (0.5 + \frac{\sin^2[\sqrt{x_1^2+x_2^2}]-0.5}{[1+0.001(x_1^2+x_2^2)]^2})$	$[-100, 100]$	40	0.5
$F_{27}(x) = -4\lvert\sin(x_1)\cos(x_2)e^{\lvert\cos((x_1^2+x_2^2)/200\rvert}\rvert$	$[-10, 10]$	10	-10.8723
$F_{28}(x) = -0.0001[\lvert\sin(x_1)\sin(x_2)e^{\lvert 100-[(x_1^2+x_2^2)^{0.5}]/\pi\rvert}\rvert + 1]^{0.1}$	$[-10, 10]$	20	0.1
$F_{29}(x) = -\exp[-\lvert\cos(x_1)\cos(x_2)e^{\lvert 1-[(x_1^2+x_2^2)^{0.5}]/\pi\rvert^{-1}}\rvert]$	$[-11, 11]$	30	-0.96354
$F_{30}(x) = \sum_{i=1}^{m-1} (-(x_{i+1}+47)\sin(\sqrt{\lvert x_{i+1}+x_i/2+47\rvert}) + sin(\sqrt{\lvert x_i - x_{i+1}+47\rvert})(-x_i))$	$[-512, 512]$	30	959.64

Table 7
The naming style for ChOAs.

Chaotic map	Updating Strategies					
	Quadratic	Gauss/Mouse	Logistic	Singer	Bernoulli	Tent
Type 1 (ChOA1)	ChOA11	ChOA12	ChOA13	ChOA14	ChOA15	ChOA16
Type 2 (ChOA2)	ChOA21	ChOA22	ChOA23	ChOA24	ChOA25	ChOA26

Table 8
Parameters and initial values of the benchmark algorithm.

Algorithm	Parameter	Value
ChOA	f	Table I
	r_1, r_2	Random
	m	Chaotic
	Number of Chimps	50
	Maximum number of iterations	250
BBO	Habitat modification probability	1
	Immigration probability bounds per gene	[0,1]
	Step size for numerical integration of probabilities	1
	Max immigration (I) and Max emigration (E)	1
	Mutation probability	0.005
	Population size	50
	Maximum number of generations	250
GWO	Number of wolf	50
	Upper bound	5
	Lower bound	−5
	Maximum number of iterations	250
LGWO	a_0	2
	β	~U(0,2)
	p	~U(0,1)
ALO	w	[2,6]
	Number of search agent	50
	Modified bound	[−100,100]
	Maximum number of iterations	250
BH	a	[0,1]
	Number of stars	100
	Maximum number of iterations	250
PSO	Cognitive constant (C_1)	1
	Social constant (C_2)	1
	Local constant (W)	0.3
	Population size	50
	Maximum number of iterations	250
GA	Type	Real coded
	Selection	Roulette wheel
	Recombination	Single-point (1)
	Mutation	Uniform (0.01)
	Layout	Full connection
	Population size	50
	Maximum number of iterations	250
GSA	Population size	50
	Number of masses	30
	Gravitational constant	1
	Maximum number of iterations	250
	pa	0.25
CS	Population size	50
	Maximum number of iterations	250

Table 9
The results of unimodal benchmark functions.

Algorithm		F_1	F_2	F_3	F_4	F_5	F_6	F_7
ChOA11	Ave	5.9216e-33	1.0792e-19	1.9616e-08	1.0878e-08	27.1256	0.78715	0.0011101
	Std	0.0000507	0.00017244	0.0015376	0.00029664	0.001221	0.00066241	0.0004353
	p-value	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ChOA12	Ave	**6.8573e-49**	**2.1821e-28**	**1.3912e-08**	**1.4402e-12**	27.1546	0.2159	**0.0011056**
	Std	**0.00000003**	**0.00000035**	**0.0000141**	**0.00001211**	0.0016241	0.00091769	**0.683e-05**
	p-value	**N/A**	**N/A**	**N/A**	**N/A**	0.0001	0.0001	**N/A**
ChOA13	Ave	5.793e-25	2.6344e-15	0.0016344	2.0177e-06	27.1812	0.59441	0.0014983
	Std	0.00044803	0.0025339	0.0008674	0.00036884	0.0012275	0.00022243	0.000892
	p-value	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ChOA14	Ave	2.2761e-07	1.337e-06	2.3752	0.029591	29.0001	0.92924	0.0020332
	Std	0.00068042	0.00028847	0.00061586	9.2999e-05	0.00070309	0.00028788	0.0012945
	p-value	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047	0.0047
ChOA15	Ave	1.7851e-09	1.0453e-07	1.9476	0.028648	28.0388	1.2473	0.0019176
	Std	0.0023071	0.00058469	0.0014968	0.00034511	0.00048538	0.0024164	0.0001202
	p-value	0.00797	0.00797	0.00797	0.00797	0.00797	0.00797	0.00797
ChOA16	Ave	2.0825e-05	9.8232e-05	2.02153	2.4511	33.9835	3.9568	0.010535
	Std	0.00088144	0.00020824	0.00048348	0.00014341	0.00076807	0.0008887	0.0009381
	p-value	0.00797	0.0047	0.0047	0.0047	0.00797	0.00797	0.0001
BBO	Ave	0.013011	0.2334	3.9745	1.7185	40.8825	0.39002	0.081996
	Std	0.00028037	0.0013183	0.00057292	0.00036264	0.00034323	0.0007505	3.456e-05
	p-value	6.39e-05	6.39e-05	0.0047	6.39e-05	0.00797	0.00797	0.0001
BH	Ave	0.27385	1.132	3.9664	1.0276	50.9993	0.065608	0.11347
	Std	0.00093598	6.3233e-05	0.0016358	0.00026678	0.0021593	0.00073555	2.793e-05
	p-value	0.00797	6.39e-05	6.39e-05	6.39e-05	0.00797	0.00797	0.0001
ALO	Ave	1.2444	2.3692	3.8894	33.7621	63.3616	2.0344	0.79273
	Std	0.00092526	7.7181e-05	0.0010751	0.0011727	0.0007689	0.00073103	0.001886
	p-value	0.0057	6.39e-05	6.39e-05	6.39e-05	6.39e-05	6.39e-05	6.39e-05
GWO	Ave	5.0555e-13	4.6933e-08	0.7213	0.00074374	28.8595	1.2504	0.0014836
	Std	0.00040636	0.00013043	0.00078211	7.9584e-06	7.3315e-05	0.00063077	0.0005685
	p-value	0.0057	0.0049	6.39e-05	6.39e-05	6.39e-05	0.0049	0.0057
ChOA21	Ave	1.6375e-26	6.8737e-17	3.8819e-07	1.4583e-06	27.1736	0.50289	0.0016598
	Std	0.00057013	0.00068177	0.00016291	0.00029769	0.00010131	0.00098584	0.0015441
	p-value	0.00797	0.0049	0.0049	0.0057	0.0049	0.0049	0.0057
ChOA22	Ave	1.552	1.1836	1.759	0.9313	58.5251	1.2803	3.3205
	Std	0.00114	0.0013475	9.319e-05	0.00077073	0.0011949	0.0019343	0.0011812
	p-value	0.00797	0.0049	0.0049	0.00797	0.0057	0.0049	0.0057
ChOA23	Ave	0.0042069	0.46036	2.06	0.7323	**20.8532**	0.058577	0.10077
	Std	0.00031728	8.4022e-05	0.0013834	0.00052112	**0.00010311**	0.00026976	7.668e-05
	p-value	0.0057	0.00797	0.0049	0.00797	**N/A**	0.0057	0.00797
ChOA24	Ave	0.43608	0.65876	2.7305	1.8589	39.7043	0.18706	0.072983
	Std	0.00087541	0.001156	0.00011975	0.0030722	0.00026654	0.0011506	0.0004988
	p-value	0.0057	0.0057	0.0049	0.0049	0.0057	0.00797	0.00797
ChOA25	Ave	0.02323	0.58268	3.3941	1.9377	40.7174	**0.0064761**	0.063093
	Std	0.00027643	0.00016033	0.0012325	0.0011574	0.00032931	**0.00000431**	0.0010881
	p-value	0.0057	0.0049	0.0049	0.0057	0.00797	**N/A**	0.00797
ChOA26	Ave	0.10797	0.16594	2.6906	1.3117	38.7458	0.070298	0.11899
	Std	0.00036936	0.00084257	0.00013504	0.00023687	0.0015624	0.0016968	0.0005133
	p-value	0.0057	0.00797	0.0049	0.0049	0.0057	0.0049	0.0049

Fig. 8 shows the process of updating the search chimp's location in two-dimensional search space regarding the position of other chimp positions. As it can be seen, the final position is located randomly in a circle which is defined by attacker, barrier, chaser and driver chimp positions. In other words, the prey position is estimated by four best groups and other chimps randomly update their positions within its vicinity.

2.2.3. *Prey attacking (utilization)*

As mentioned previously, in the final stage, the chimps will attack the pray and finish the hunt as soon as the prey stops moving. To mathematically model the attacking process, the value of f should be reduced. Note that the variation range of the $\mathbf{a}$ is also reduced by $\mathbf{f}$. In other words, $\mathbf{a}$ is a random variable in the interval of $[-2\mathbf{f},2\mathbf{f}]$, whereas the value of $\mathbf{f}$ reduces from 2.5 to 0 in the period of iterations. When the random values of a lie in the range of $[-1,1]$, the next position of a chimp can be in any location between its current position and the position of the prey. Fig. 9 shows that the inequality forces the chimps to attack the prey.

According to the operators that have already been presented, ChOA allows the chimps to update their positions according to the positions of attacker, barrier, chaser, and driver chimps and attack the prey. However, ChOAs may still be at the risk of trapping in local minima, so other operators are required to avoid this issue. Although, the proposed driving, blocking, and chasing mechanism somehow shows exploration process, ChOA requires more operators to emphasize exploration phase.

2.2.4. *Searching for pray (exploration)*

As previously mentioned, the exploration process among the chimps is mainly done considering the location of attacker, barrier, chaser, and driver chimps. They diverge to seek for the prey and aggregate to attack prey. In order to mathematically model the divergence behaviour, the $\mathbf{a}$ vector with a random value bigger

Table 10
The results of multimodal benchmark functions.

Algorithm		F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}
ChOA11	Ave	−6432.073	**5.6843e-14**	3.9968e-14	**0**	0.03789	0.59045
	Std	3.2605	**0.0007579**	0.014542	**0**	0.00062651	0.018694
	p-value	0.0001	**N/A**	0.0057	0.0057	0.0049	0.0049
ChOA12	Ave	−3150.5985	2.738	7.9936e-15	**0**	0.29035	1.7768
	Std	21.9845	0.008412	0.000851	**0**	0.015873	0.0097053
	p-value	0.0057	0.0057	0.0049	0.0049	0.0001	0.0001
ChOA13	Ave	−3628.8022	**5.6843e-14**	1.0036e-13	**0**	0.043508	**0.53169**
	Std	5.1249	0.0012031	0.0051235	**0**	0.010383	**0.003692**
	p-value	0.0057	0.0057	0.0049	0.0049	0.0001	**N/A**
ChOA14	Ave	−5652.3897	1.1596	2.9619	0.036661	0.16291	3.0763
	Std	7.6746	0.0030101	0.010755	0.0015965	0.014936	0.018627
	p-value			0.0057	0.0057	0.0049	0.0049
ChOA15	Ave	−5594.9085	1.3417	2.9668	0.014122	0.07438	2.8352
	Std	4.5861	0.0010454	0.0075506	0.013896	0.00086557	0.019978
	p-value	0.00747	0.0001	0.0057	0.0057	0.0049	0.0049
ChOA16	Ave	−5588.3064	5.5591e-05	2.9668	0.42516	0.4587	2.2013
	Std	21.4912	0.0064053	0.013362	0.0076654	0.007336	0.018548
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049	0.0001
BBO	Ave	−9952.6876	1.0027	0.5129	0.29361	2.1009	4.8418
	Std	7.2952	0.012332	0.01008	0.0074866	0.0089069	0.0047195
	p-value	6.39e-05	6.39e-05	6.39e-05	0.0057	0.0057	0.0049
BH	Ave	−9230.6623	2.0007	0.6284	0.31144	1.1078	0.39991
	Std	5.5351	0.017319	0.011713	0.0096344	0.014307	2.8186e-05
	p-value	6.38e-05	6.39e-05	6.39e-05	6.39e-05	6.38e-05	6.38e-05
ALO	Ave	−7167.2698	2.4653	0.8945	2.5676	1.5463	5.7996
	Std	10.5861	0.012509	0.0092374	0.016871	0.0043937	0.00586957
	p-value	6.38e-05	6.38e-05	6.39e-05	6.39e-05	6.39e-05	0.0001
GWO	Ave	−5665.3886	1.4001	2.9667	0.014955	0.081592	2.316
	Std	24.838	0.0066279	0.0073361	0.0084009	0.0056174	0.010415
	p-value	6.39e-05	6.39e-05	6.39e-05	6.39e-05	6.39e-05	0.0001
ChOA21	Ave	−6738.8454	0.0026	**1.2168e-15**	**0**	**0.027962**	0.8424
	Std	6.4067	0.010056	**0.0000505**	**0**	**0.0000267**	0.023382
	p-value	6.39e-05	6.39e-05	**N/A**	**N/A**	**N/A**	0.0045
ChOA22	Ave	−2609.1446	2.2198	0.7874	0.13063	0.1821	**0.52586**
	Std	3.81561	0.013331	0.00065341	0.0036728	0.0076626	**0.000825**
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049	**N/A**
ChOA23	Ave	**−1023.9291**	1.0698	1.2264	0.12105	3.0047	2.513
	Std	**1.8642**	0.0032444	0.010339	0.016238	0.0046195	0.0013249
	p-value	**N/A**	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA24	Ave	−9655.5997	1.42	0.7238	0.010202	0.283	5.1047
	Std	10.4246	0.0070324	0.002375	0.011905	0.016156	0.02809
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049	0.0001
ChOA25	Ave	−9146.7686	1.6997	1.7298	0.025172	7.4374	4.3983
	Std	9.9677	0.017173	0.014912	0.010107	0.013853	0.0070056
	p-value	0.0001	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA26	Ave	−9026.9166	1.7645	1.3173	0.01028	0.89062	3.9773
	Std	4.356	0.006938	0.014907	0.013683	0.00010712	0.00455227
	p-value	0.00747	0.00747	0.00747	0.0057	0.0057	0.0049

than 1 or smaller than −1 is used, so that the search agents are forced to diverge and get distant from prey. This procedure shows the exploration process and allows the ChOA to search globally. Fig. 9 shows that the inequality $|\mathbf{a}| > 1$ forces the chimps to scatter in the environment to find a better prey. This section is inspired from GWO (Mirjalili, 2013).

Another ChOA component that affects the exploration phase is the value of **c**. As in Eq. (4), **c** vector elements are random variables in the interval of [0,2]. This component provides random weights for prey to reinforce ($\boldsymbol{c} > 1$) or lessen ($\boldsymbol{c} < 1$) the effect of prey location in the determination of the distance in Eq. (5). It also helps ChOA to enhance its stochastic behaviour along the optimization process and reduce the chance of trapping in local minima. **c** is always needed to generate the random values and execute the exploration process not only in the initial iterations, but also in the final iterations. This factor is very useful for avoiding local minima, especially in the final iterations. **c** vector is also considered as the influence of the obstacles which prevent chimps from approaching the prey in nature. In general, natural obstacles in the path of chimps prevent them from approaching the prey with proper speed. This is the precise expression of the **c** vector effect. Depending on chimp's position, the **c** vector can assign a random weight to prey in order to make the hunt harder or easier.

2.2.5. Social incentive (sexual motivation)

As mentioned previously, acquiring meet and subsequent social motivation (sex and grooming) in the final stage causes chimps to release their hunting responsibilities. Therefore, they try to obtain meat forcefully chaotic.

This chaotic behaviour in final stage helps chimps to further alleviate the two problems of entrapment in local optima and slow convergence rate in solving high-dimensional problems.

Table 11
The results of fixed-dimension multimodal benchmark functions.

Algorithm		F_{14}	F_{15}	F_{16}	F_{17}	F_{18}
ChOA11	Ave	0.998	0.020364	−1.0316	0.39792	3
	Std	0.00059718	0.010885	0.0095395	0.0047829	0.010803
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA12	Ave	0.998	**0.00034398**	−1.0316	0.39792	3
	Std	0.0057686	**0.00000001**	0.021147	0.00026104	0.009066
	p-value	0.00747	**N/A**	0.0057	0.0049	0.0049
ChOA13	Ave	0.99801	0.00067708	−1.0316	0.39865	3.0001
	Std	0.0047493	0.0068029	0.0096524	0.015434	0.03319
	p-value	0.0001	0.0001	0.00747	0.0057	0.0057
ChOA14	Ave	0.998	0.0012896	−1.0316	0.39833	3.0002
	Std	0.00037144	0.011627	0.0052976	0.0020581	0.0067462
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA15	Ave	0.99802	0.00125	−1.0316	0.39796	3.0001
	Std	0.014758	0.0077204	0.016366	0.0023049	0.00043324
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA16	Ave	0.99809	0.0013562	−1.0316	0.39805	3.0001
	Std	0.0024767	0.0011969	0.00089862	0.0062243	0.0070154
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
BBO	Ave	0.998	0.00042263	−1.0316	0.39789	3
	Std	0.0090912	0.01693	0.014821	0.0065533	0.01335
	p-value		6.39e-05	6.39e-05	6.39e-05	6.39e-05
BH	Ave	0.998	0.020363	−1.0316	0.39789	3
	Std	0.0038131	0.00066429	0.012439	0.014056	0.021488
	p-value	6.39e-05	6.39e-05	6.39e-05	6.39e-05	6.39e-05
ALO	Ave	0.998	0.020363	−1.0316	0.39789	3
	Std	0.021358	0.0017314	0.012602	0.0096567	0.007996
	p-value	6.39e-05	6.39e-05	6.39e-05	6.39e-05	6.39e-05
GWO	Ave	0.998	0.0012849	−1.0316	0.39842	3.0001
	Std	0.018609	0.020989	0.021182	0.0097607	0.0099344
	p-value	6.38e-05	6.39e-05	6.38e-05	6.39e-05	6.38e-05
ChOA21	Ave	0.998	0.00035113	−1.0316	0.39789	3.0004
	Std	0.0063754	0.0028921	0.0011222	0.013257	0.011011
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA22	Ave	2.9821	0.00099085	−1.0316	0.39789	3
	Std	0.0078901	0.0042495	0.010545	0.00093272	0.016413
	p-value	0.00747	0.0057	0.0057	0.0049	0.0001
ChOA23	Ave	**0.998**	**0.00034349**	**−1.0316**	**0.39789**	**3**
	Std	**0.0001241**	**0.00000013**	**0.0000741**	**0.000793**	**0.0001531**
	p-value	**N/A**	**N/A**	**N/A**	**N/A**	**N/A**
ChOA24	Ave	0.998	0.00033082	−1.0316	0.39789	3
	Std	0.0092874	0.0071116	0.0061951	0.0045872	0.01198
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA25	Ave	0.998	0.0016554	−1.0316	0.39789	3
	Std	0.0056774	0.006215	0.0010265	0.0086782	0.0020994
	p-value	0.0047	0.00757	0.0057	0.0049	0.0049
ChOA26	Ave	0.998	0.00030769	−1.0316	0.39789	3
	Std	0.010431	0.0032917	0.0047039	0.0044647	0.00028328
	p-value	0.00547	0.00457	0.0057	0.0049	0.0049

The chaotic maps which have been used to improve the performance of ChOA are explained in this section. Six chaotic maps have been used in this article as shown in Table 2 and Fig. 10. These chaotic maps are deterministic processes which also have random behaviour. In this article, value 0.7 has been considered as the primary point of all the maps in accordance with reference (Saremi, Mirjalili, & Lewis, 2014). To model this simultaneous behaviour, we assume that there is a probability of 50% to choose between either the normal updating position mechanism or the chaotic model to update the position of chimps during optimization. The mathematical model is expressed by Eq. (9).

$$\mathbf{x}_{chimp}(t+1) = \begin{cases} \mathbf{x}_{\text{prey}}(t) - \mathbf{a}.\mathbf{d} if \mu < 0.5 \\ \textbf{Chaotic\_value} if \mu \geq 0.5 \end{cases} \quad (9)$$

Where μ is a random number in [0,1].

In brief, the searching process in ChOA begins with generating a stochastic population of chimps (candidate solutions). Then, all chimps are randomly divided into four predefined independent groups entitled attacker, barrier, chaser and driver. Each chimp updates its **f** coefficients using the group strategy. During the iteration period, attacker, barrier, chaser and driver chimps estimate the possible prey locations. Each candidate solution updates its distance from the prey. Adaptive tuning the **c** and **m** vectors cause local optima avoidance and faster convergence curve, simultaneously. The value of **f** is reduced from 2.5 to zero, to enhance the process of exploitation and attacking the prey. The inequality $|\mathbf{a}| > 1$ results in divergence of the candidate solutions, otherwise, they eventually converge toward the prey. Fig. 11 presents the pseudocode of ChOA.

Table 12
The results of fixed-dimension multimodal benchmark functions (continued).

Algorithm		F_{19}	F_{20}	F_{21}	F_{22}	F_{23}
ChOA11	Ave	−3.8622	−3.1969	−2.593	**−10.2537**	−9.3837
	Std	0.019162	0.018342	0.022841	0.014715	0.0066907
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA12	Ave	−3.8619	−3.1825	−6.7593	−9.2651	−7.9056
	Std	0.0071951	0.00030283	0.0070818	0.0043589	0.0081338
	p-value	0.001	0.00747	0.0057	0.0057	0.0001
ChOA13	Ave	−3.8614	−3.3106	−7.9664	−8.6936	−10.0206
	Std	0.012393	0.0093514	0.016142	0.0085863	0.0021619
	p-value	0.00452	0.00747	0.0057	0.0057	0.0049
ChOA14	Ave	−3.8547	**−2.0591**	−4.8606	−5.0383	−5.0358
	Std	0.006271	**0.0001319**	0.01722	0.0076048	0.013176
	p-value	0.0045	**N/A**	0.00747	0.0057	0.0057
ChOA15	Ave	−3.8548	−2.6329	−5.0189	−0.91158	−5.1029
	Std	0.021395	0.014033	0.013209	0.0041865	0.0017119
	p-value	0.0001	0.00747	0.0057	0.0057	0.0049
ChOA16	Ave	−3.8624	−2.2492	−4.8537	−4.8401	−4.8898
	Std	0.00026981	0.0075036	0.00034583	0.008309	0.0075659
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
BBO	Ave	−3.8628	−3.322	−5.1008	−2.7519	−10.5364
	Std	0.0051	0.0044809	0.010935	0.014868	0.014644
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05
BH	Ave	−3.8628	−3.2031	−2.6829	−10.4029	−10.5364
	Std	0.0034586	0.00073532	0.0091199	0.0021605	0.022491
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05
ALO	Ave	−3.8628	−3.1974	−10.1532	−10.4029	−10.5364
	Std	0.0077814	0.0075256	0.0051117	0.0067976	0.016077
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05
GWO	Ave	−3.8541	−3.0731	−0.88288	−5.0532	−5.0601
	Std	0.0027469	0.00016015	0.0067064	0.0016984	0.0034639
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05
ChOA21	Ave	−3.8627	−3.322	−10.1505	−10.4028	−10.5336
	Std	0.011721	0.011144	0.003524	0.0017081	0.0044508
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA22	Ave	−3.8628	−3.322	−2.6829	**−10.4029**	−10.5364
	Std	0.0053146	0.00084038	0.0039454	**0.0004916**	0.0018552
	p-value	0.0045	0.00747	0.0057	**N/A**	0.0001
ChOA23	Ave	**−3.8628**	**−3.2031**	**−10.1532**	−2.7659	**−10.5364**
	Std	**0.000045**	**0.0001641**	**0.0000534**	0.0035001	**0.0015218**
	p-value	**N/A**	**N/A**	**N/A**		**N/A**
ChOA24	Ave	−3.8628	−3.322	−5.1008	−10.4029	−10.5364
	Std	0.013575	0.0039439	0.010463	9.5041e-05	0.022171
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA25	Ave	−3.8628	−3.2031	−10.1532	−10.4029	−10.5364
	Std	0.003998	0.011924	0.0079751	8.3701e-05	0.0023049
	p-value	0.00747	0.0057	0.0057	0.0049	0.0049
ChOA26	Ave	−3.8628	−3.322	−10.1532	−10.4029	−10.5364
	Std	0.012586	0.01132	0.018268	0.02068	0.012983
	p-value	0.0045	0.00747	0.0057	0.0057	0.0049

3. Simulation results and discussion

In this section, the ChOA is tested on 30 benchmark functions. The first 23 test functions are the classical benchmark functions used in the many kinds of research (Digalakis & Margaritis, 2001; Molga & Smutnicki, 2005; Yang, 2010). Generally, these functions are divided into three groups: unimodal, multimodal, and fixed-dimension multimodal which are reported in Tables 3–5, respectively. In these tables, dim indicates the dimension of the problem, fmin is the minimum reported in the literature, and Range is the boundary of the problem's search space.

The three aforementioned groups of benchmark functions are utilized with different characteristics to test the performance of the ChOA from different aspects. As their names imply, unimodal benchmark functions have a single minimum so they can test the exploitation and convergence rate of ChOA. In contrast to unimodal, multimodal benchmark functions have more than one minimum, making them more challenging than unimodal benchmarks. Therefore, exploration and local minima avoidance of optimizers can be tested by the multimodal benchmark functions. It should be mentioned that the difference between fixed-dimensional multimodal benchmarks in Table 4 and multimodal benchmarks in Table 5 is the ability to define the desired number of design variables. The mathematical models of fixed-dimensional benchmark functions do not let us tune the number of design variables, but they prepare various search space compared to multimodal benchmark functions in Table 4.

In the following, in order to have a comprehensive comparison, we use other newly proposed rotated and shifted benchmark function defined in the IEEE CEC 2013 special session and Competition on Niching Methods for Multimodal Function Optimization (Li, Engelbrecht, & Epitropakis, 2013) and also (Mishra, 2007). The

Table 13
The results of shifted and rotated benchmark functions.

Algorithm		F_{24}	F_{25}	F_{26}	F_{27}	F_{28}	F_{29}	F_{30}
ChOA11	Ave	−104.6332	2.0711e-5	1.9616e-9	−8.2221	0.9256	−0.78715	1000.254
	Std	0.0000607	0.0011244	0.0001376	0.000114	0.001221	0.00054	10.2254
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.00747
ChOA12	Ave	−103.2584	1.1821e-4	1.4412e-8	−7.0022	0.9946	−0.5214	999.547
	Std	0.004101	0.0000122	0.0001457	0.000012	0.00162	0.00091	11.2547
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.00747
ChOA13	Ave	−102.2589	2.6224e-5	2.0041e-7	−7.1245	0.8812	−0.59441	1001.0254
	Std	0.0022803	0.0021139	0.005574	0.00022	0.0099275	0.00044	12.2547
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.00747
ChOA14	Ave	−102.2369	0.0012	1.22335	−6.029591	0.8001	10.9291	985.635
	Std	0.068042	0.0022847	0.00001586	9.2999e-05	0.000703	0.00028	9.2514
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.00747
ChOA15	Ave	−102.3251	1.0453e-4	1.33254	−6.0286	1.0888	10.2473	998.254
	Std	0.03071	0.001845	0.0014448	0.00011	0.000485	0.00241	2.2547
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.00747
ChOA16	Ave	−102.3214	0.0058	3.12451	−6.0011	1.0005	15.9568	1000.0214
	Std	0.000224	0.00980824	0.00048348	0.000121	0.00076	0.00089	5.5454
	p-value	0.007937	0.0001	0.0001	0.0047	0.0047	0.0001	0.0047
BBO	Ave	−100.8888	3.2584	4.9745	−5.7122	1.8825	2.39002	1005.888
	Std	0.0002803	0.01183	0.0005112	0.00364	0.000343	0.0007505	12.547
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05	6.39e-05	6.39e-05
BH	Ave	−99.2233	4.2154	4.1214	−5.3621	1.0093	1.065608	1009.214
	Std	0.002298	0.001254	0.0011118	0.00026	0.00215	0.000735	14.2147
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00005	6.39e-05	6.39e-05	6.39e-05
ALO	Ave	−100.2589	5.2104	3.2147	−6.3254	1.3644	8.0344	999.357
	Std	0.0011126	0.000114	0.0020711	0.00117	0.0007689	0.00073	3.2546
	p-value	6.38e-05	6.39e-05	6.38e-05	0.0001	6.39e-05	6.39e-05	6.39e-05
GWO	Ave	−102.9999	0.14572	0.00013	−6.00074	1.8595	10.2504	984.2145
	Std	0.0002236	0.0002243	0.001111	0.000123	7.3315e-05	0.000639	4.5454
	p-value	6.38e-05	6.39e-05	6.38e-05	0.00047	6.39e-05	6.39e-05	6.39e-05
ChOA21	Ave	−104.2312	1.8737e-17	**0.004e-09**	−7.2145	**0.1006**	**−0.50289**	968.245
	Std	0.0001013	0.0002177	**9.319e-05**	0.000297	**0.000007**	**0.00098584**	3.2458
	p-value	0.007937	**0.4429**	**N/A**	0.0081	**N/A**	**N/A**	0.0047
ChOA22	Ave	**−105.2587**	**0.1245e-20**	3.8819e-07	**−7.5214**	21.5251	−0.2803	**960.999**
	Std	**0.0000114**	**0.0000075**	0.00088791	**0.0000707**	5.22119	0.0019343	**1.25478**
	p-value	**N/A**	**N/A**	0.0047	**N/A**	6.39e-05	0.04785	**N/A**
ChOA23	Ave	−104.8555	0.00036	3.7777	−6.1323	0.8532	1.058577	987.245
	Std	0.0003172	8.4022e-05	0.0013834	0.00012	0.000125	0.00026976	3.6547
	p-value	0.007937	0.0001	0.0001	0.00014	0.0074	0.0001	0.0047
ChOA24	Ave	−103.2555	0.00876	4.2305	−7.0009	0.7043	2.18706	984.258
	Std	0.0012541	0.001156	0.0001175	0.00307	0.000266	0.0011506	5.2145
	p-value	0.007937	0.0001	0.0001	0.00047	0.0047	0.00047	0.00747
ChOA25	Ave	−103.2411	1.58268	5.3941	−6.9377	159.7174	3.0064761	982.021
	Std	0.0017643	0.0011603	0.0012325	0.00174	0.00032931	0.0008431	4.25896
	p-value	0.007937	0.0001	0.0001	0.000041	0.0001	0.0047	0.00747
ChOA26	Ave	−102.0541	0.16594	4.2211	−6.3117	111.7458	2.088298	1000.024
	Std	0.0016936	0.003257	0.00013504	0.000287	0.0015624	0.00169	7.2145
	p-value	0.007937	0.0001	0.00098	0.00021	0.0001	0.0001	0.00747

remaining benchmark functions are more complex and follow the paradigm of composition functions. The mathematical models of these benchmark functions are shown in Table 6.

The ChOAs have been divided into and named with regard to the type of the dynamic strategies for independent groups (illustrated in Table 1) and the number of the chaotic maps (illustrated in Table 2). For instance, if the dynamic strategy number one-(from Table 1) and tent map (from Table 2) has been used to enhance ChOA, the name of that algorithm will be ChOA16 in such a way that 1 refers to the dynamic strategy type 1 and 6 refers to the row number of the temp map in Table 2. This type of naming has been thoroughly shown in Table 7.

Figs. 12 to 15 draw a comparison between the convergence curves of different algorithms for unimodal, multimodal, fixed-dimension multimodal, and rotated and shifted benchmark functions, respectively.

For verifying the results, the ChOAs are compared to ALO (Mirjalili, 2015) as a kind of SIAs, BBO (Simon, 2008) as a powerful kind of EAs and BH (Hatamlou, 2013) as a physics-based algorithm. In addition, the ChOAs are compared with GWO (Mirjalili et al., 2014) as the most famous hunting-based benchmark algorithm. The parameters of these algorithms are presented in Table 8.

For these experiments, each test is carried out a Windows 10 system using Intel Core i7, 3.8 GHz, 16 G RAM and, Matlab R2016a. The ChOA algorithms were run 30 times on each benchmark function. The statistical results (Average (Ave), Standard Deviation (Std), and *p*-value) are reported in Tables 9 to 13. The best results are illustrated in bold type. The concepts of Ave and Std can be used to show the algorithms capability of avoiding the local minima. The lower the value of Ave, the greater the algorithms capability of finding a solution near the global optimum. Although the Ave value

Table 14
The results of unimodal benchmark functions (100-dimensional).

Function	ChOA		PSO		GSA		BH	
	Ave	Std	Ave	Std	Ave	Std	Ave	Std
F1	**2.8e-09**	**1.1e-09**	2.799	0.721	86.213	0.04243	9.34	2.0731
F2	44.16	18.17	23.87	2.432	152.44	0.00122	320.82	38.28
F3	**194.11**	**42.18**	391.34	41.57	169.88	0.03780	900.75	409.06
F4	**2.18**	**0.0846**	3.131	0.0879	10.265	0.0013	5.6670	0.8293
F5	13.94	1.71	75.2342	5.245	321.13	0.05329	117.80	49.07
F6	**1.60e-07**	**1.09e-08**	3.421	1.206	207.13	0.0003	4.2056	1.005
F7	**0.000546**	**0.004407**	1.721	4.0133	2.309	1.99e-05	0.4344	0.0127
	GWO		CS		LGWO		GA	
	Ave	Std	Ave	Std	Ave	Std	Ave	Std
F1	4. 989	2. 678	3.80e-05	1.85e-05	6.1218	0.5744	18.7475	15.500
F2	28.647	0.9384	33.10	0.0656	**22.191**	**1.0191**	526.612	92.127
F3	2219	85.540	1,9527	52.7511	1852	418.4	933.21	332.1
F4	3.689	0.4572	2.9362	0.6877	2.736	0.0947	8.5144	0.5321
F5	262.7	11.490	27.672	13.88	**11.171**	**0.76**	91.4449	62.706
F6	13.99	2.1091	1.17e-05	1.55e-05	1.11e-05	1.05e-05	40.56	23.681
F7	0.8391	0.05354	0.00131	0.0087	0.00273	0.00041	9.56	5.1061

of the two algorithms can be equal, their performance in finding the global optimum may differ in each iteration. Thus, Std is used to make a better comparison. To have a lower dispersion of results, the Std should have a small value.

According to Derrac, García, Molina, and Herrera (2011), statistical tests are required to evaluate the performance of MOAs adequately. Comparing MOAs according to their Ave and Std values is not enough (Garcia, Molina, Lozano, & Herrera, 2009) and a statistical test is needed to indicate a remarkable improvement of a new MOA in comparison to the other existing MOAs to solve a particular optimization problem (Mirjalili & Lewis, 2013). In order to see whether the results of ChOA differ from other benchmark algorithms in a statistically significant way, Wilcoxon's rank-sum (Wilcoxon, 1945), which is a non-parametric statistical test, was performed and the significance level of 5% accomplished. The calculated *p*-values of the Wilcoxon's rank-sum are given in the results as well. The N/A in tables is the abbreviation of "Not Applicable" which means that the corresponding MOA cannot be compared with itself in the rank-sum test. Conventionally, *p*-values less than 0.05 are considered as strong evidence against the null hypothesis. Note that *p*-values greater than 0.05 are underlined in the tables.

3.1. Evaluation of exploitation ability

Functions F1, F2,..., F7 have only one global optimum since they are unimodal. These benchmark functions permit to evaluate the exploitation capability of the investigated MOAs. Table 9 illustrates ChOAs is very competitive with other MOAs. It can be seen from Table 9 that ChOA12 has the best results in five out of seven unimodal test functions.

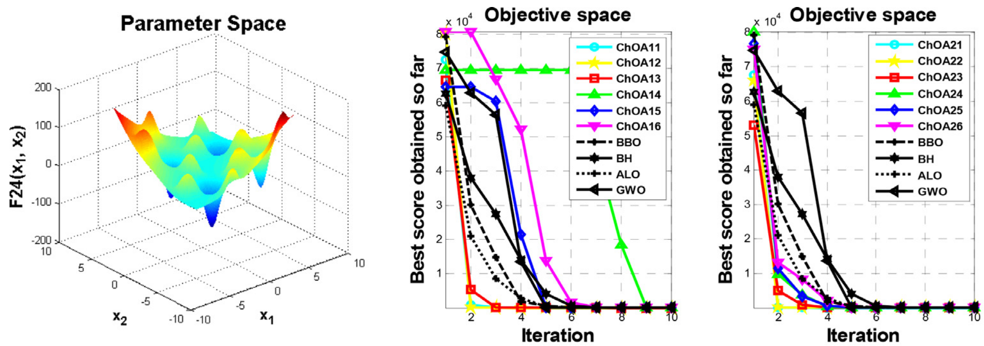
Fig. 12 shows the convergence curves of the algorithms. As can be seen from these curves, ChOA12 has the best convergence rates for most of the benchmark functions, followed by ChOA11 and ChOA21.

It is worth mentioning that unimodal test functions have no local minima and there is only one global minimum in the search space. So these kinds of benchmark functions are quite appropriate for evaluating the convergence capability of MOAs. Consequently, the results of the ChOAs show that independent groups could improve significantly the convergence ability of the ChOAs. The two main reasons for the superior results is that the chimps have diversity in their fission-fusion societies and are able to exploit knowledge of the position of near optimal solutions effectively and they also utilize the chaotic maps biasing chimps to move quickly toward the global optimum (prey). As can be seen from Fig. 12 and Table 9, among the two proposed group sets, the first group set indicates much superior results for unimodal benchmark function. This better result can be well justified by Fig. 6. According to this figure, ChOA1 has an excellent local search ability because the forms of updating strategies were chosen in such a way that different groups tend to converge faster than ChOA2 and they search more locally than globally. In other words, the reduction rate of the independent groups' coefficient of ChOA1 is faster than those coefficients of ChOA2. So, ChOA1 allows chimps to discover the search space more locally than globally, because the amplitude of the searching coefficient decreases severely after almost one-quarters of the allowed iterations.

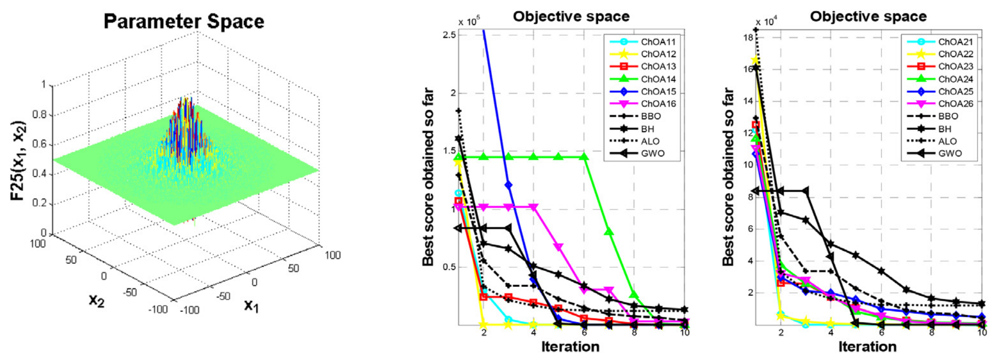
It should be noted that this considerable improvement has not been made only with categorizing chimps to independent groups but also by utilizing the new chaotic map in such a way that this chaotic behaviour in final stage helps chimps to further decline the problems of slow convergence rate. As can be seen from Fig. 12 and Table 4, the chaotic map number two i.e., Gauss/mouse map has the most significant effect on the global minima finding and convergence speed so that ChOA12 has the best results in the five out of seven unimodal benchmark functions and at least the second best optimizer in other benchmark functions. The aforementioned algorithm can hence provide fair exploitation ability.

These superior results of ChOA12 are based on the special form of the Gauss/mouse map. This chaotic map has very special shape in such a way that in early stage it has large and extremely variable amplitude, while its amplitude and variableness decrease severely in the final stages. This special shape of Gauss/mouse map causes chimps to behave both very extensively in early stage and in focus of the final stages. Generally speaking, chaotic maps provide soft transition between global and local search ability. These maps prevent chimps from quickly becoming trapped in local minima because chimps have stochastically movement even in the final stages. This stochastic movement in the final stage may be considered as sexual motivations. This is the main reason for the superior results of the proposed maps (specially the Gauss/mouse map).

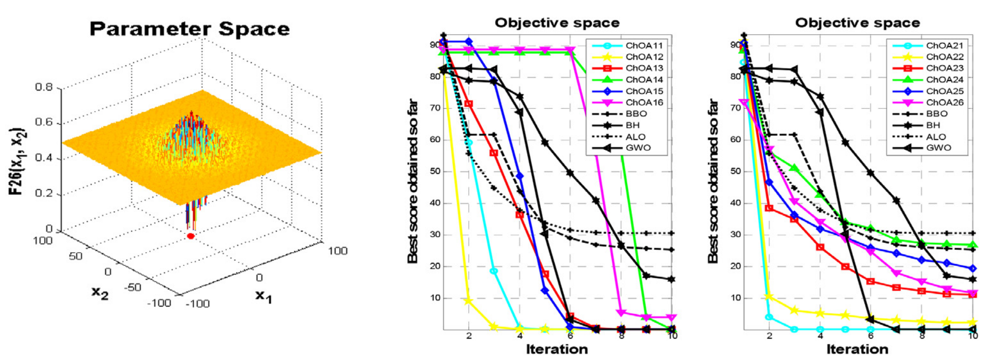
In this way, chimps tend to broadly discover promising regions of search space and exploit the best one. Chimps change abruptly in the early stages of the hunting process and then gradually converge. However, there is no additional computational cost for the proposed algorithm.



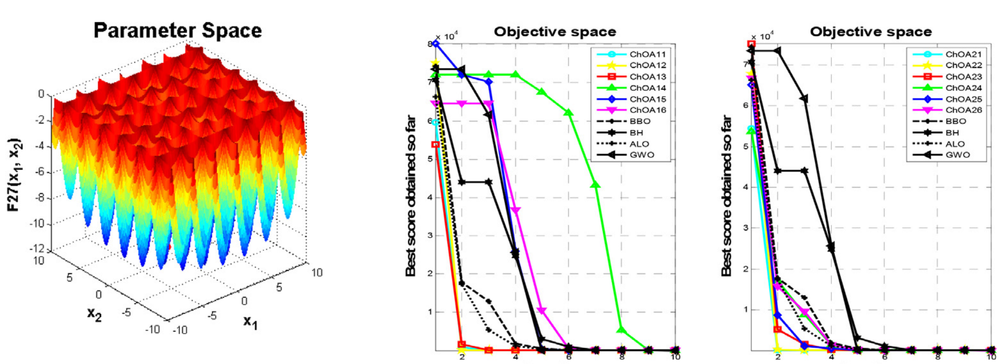
(k) F_{24}



(L) F_{25}

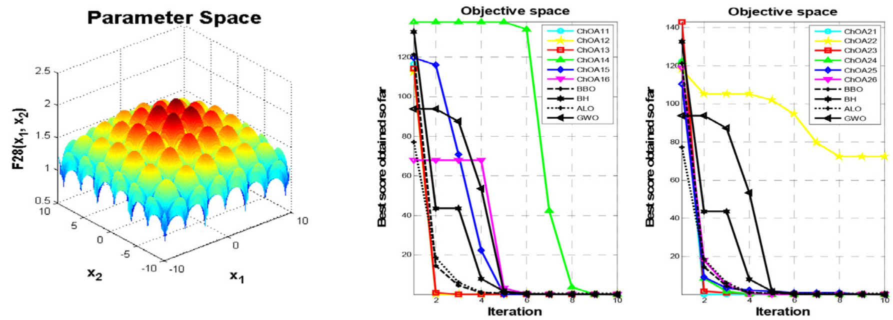


(M) F_{26}

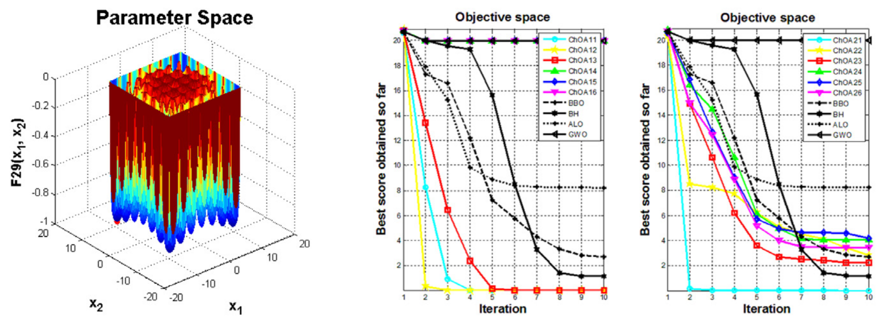


(N) F_{27}

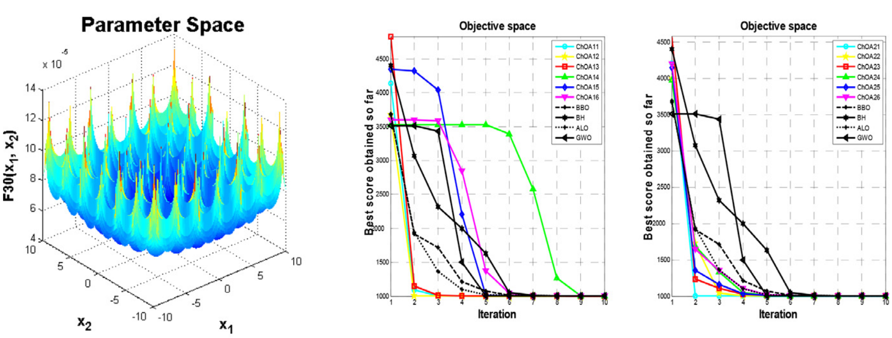
Fig. 15. Convergence curve of algorithms on the rotated and shifted multimodal benchmark functions.



(O) F_{28}



(P) F_{29}



(Q) F_{30}

Fig. 15. Continued

Table 15
The results of multimodal benchmark functions (100-dimensional).

p	ChOA		PSO		GSA		BH	
	Ave	Std	Ave	Std	Ave	Std	Ave	Std
F8	**−44,426**	**144.5**	−18,136	4962.4	−35,969	1876	−25,632	869.47
F9	**11.89**	**1.005**	62.58	2.301	12.01	0.12365	60.38	7.96
F10	**0.3058**	**0.00542**	1.183	0.07627	1.293	0.0974	1.159	0.0077
F11	0.0014	0.00021	270.2	11.49	400.5	0.8532	411	22.42
F12	**0.3982**	**0.09591**	2.07e + 05	2.77e + 05	1.00e + 08	1.99e+05	1.09e + 09	2.28e + 08
F13	**0.13915**	**0.22199**	1.24e + 06	3.82e + 05	1.00e + 08	1.99e + 05	1.25e + 09	3.85e + 08

	GWO		CS		LGWO		GA	
	Ave	Std	Ave	Std	Ave	Std	Ave	Std
F8	−55,771	3097.8	−52,600	156.04	−39,753	649.69	−28,660	1011
F9	58.95	6.653	45.58	7.889	13.45	1.058	137.8	3.155
F10	1.544	0.06684	17.654	2.982	2.4297	0.038545	1.361	0.0421
F11	**0.0011**	**0.00011**	0.001191	0.001148	1.7048	0.014301	175.8	7.3
F12	2.37e + 07	1.22e + 07	1.00e + 10	0.0045	2.426	0.05985	3.14e + 09	2.54e + 08
F13	3.87e + 07	1.80e + 07	1.00e + 10	0.0568	0.0014	0.0568	1.38e + 10	1.45e + 09

Table 16
The general description of real-world constrained optimization problem. *D* is the total number of decision variables of the problem, *g* is the number of inequality constraints and *h* is the number of equality constraints.

No	ID	Problems	D	g	h
1	RC01	Heat Exchanger Network Design (case 1)	9	0	8
2	RC04	Reactor Network Design (RND)	6	1	4
3	RC11	Two-reactor Problem	7	4	4
4	RC14	Multi-product batch plant	10	10	0
5	RC16	Optimal Design of Industrial refrigeration System	14	15	0
6	RC23	Optimal Design of Industrial refrigeration System	5	8	3
7	RC35	Optimal Sizing of Distributed Generation for Active Power Loss Minimization	153	0	148
8	RC37	Optimal Power flow (Minimization of Active Power Loss)	126	0	116
9	RC45	SOPWM for 3-level Inverters	25	24	1
10	RC51	Beef Cattle(case 1)	59	14	1

3.2. Evaluation of exploration ability

In contrast to unimodal benchmark functions, multimodal problems include many local minima whose number increases drastically with the number of design variables (problem size). Hence, this kind of benchmark functions turns very helpful if the intention is to evaluate the exploration capability of a MOA and avoiding local minima. Table 10 and Fig. 13 show the results for multimodal benchmark functions (F8-F13). As the results show, ChOAs have also fair exploration capability. In fact, ChOAs always is the most efficient algorithm in the all of the benchmark problems. This is due to the four different mechanisms of exploration in ChOAs leading this algorithm into the global optimum in the early stages and chaotic mechanism guaranteeing to reach the best result in the final stages.

The results of Table 10 indicate that the independent groups increased the performance of ChOA in terms of avoiding local minima. As may be observed in Fig. 13, similar to the results of unimodal benchmark functions, the convergence speed of ChOAs is almost better than the other MOAs. The ChOA group set 2 (especially ChOA21) have the best convergence rates among the ChOAs. The group set 2 has the special updating coefficients so that these special updating forms give ChOA more randomized search ability in comparison with ChOA group set 1; therefore, the chimps are not easily trapped in local minima. In other words, the reduction rate of the independent groups' coefficient of ChOA2 is less than those coefficients of ChOA1. As a result, the updating coefficients of ChOA2s allows chimps to discover globally the search space. Because the amplitudes of the searching coefficient decrease gradually after almost three-quarters of the allowed iterations.

In addition, the ChOA21 outperform other ChOA2s through its special chaotic map (Quadratic map). In this map, a very slight change of the input value can lead to significantly various behaviour of the map's amplitude. This particular behaviour of the quadratic map causes chimps to explore the search space completely even in the final stages in such a way that this map enhances the exploration capability of ChOA21 more than other ChOAs combined with the other chaotic map.

Unlike the multimodal test functions, the fixed-dimension multimodal benchmark functions have few local minima. As shown in Tables 11 and 12, the results of all MOAs are similar on six of the functions. However, the ChOA outperform the other MOAs on F20 to F22. ChOA23 has the best results in the almost all of these benchmark functions. Fig. 14 shows the convergence rate of the algorithms dealing with fixed-dimension benchmark functions. All the MOAs have close convergence curves, somewhat better for the ChOAs. The analogy of results and convergence rate is owing to the low dimensional characteristic of these benchmark problems; the effect of independent groups is more apparent for the high dimensional problems. To sum up, the results indicate that the independent groups and the chaotic maps are profitable for ChOA in terms not only of avoiding local optima but also improved convergence speed. The results of the ChOA show that the proposed independent groups permit chimps to have various patterns for following the social behaviour of the whole society, resulting in higher local minima avoidance ability.

Finally, for comprehensive comparison, newly proposed ChOAs were tested using some complex, rotated and shifted version of multimodal benchmark functions.

Fig. 15 and Table 13 show the results obtained using aforementioned algorithms and benchmark functions. As can be seen, the convergence rate of ChOAs were significantly better than for other MOAs, which is even better than results obtained in previous tests (unimodal, fixed-dimension and multimodal). Because the complexity of these benchmark function is more than other benchmarks. Therefore, the ability of ChOAs in these complex problem is more evident than other experiment.

3.3. Optimization of high-dimensional problems using ChOA

To further confirm the capability of ChOA in working with high-dimensional problems, this subsection investigates the 100-dimensional versions of the unimodal (F1 to F7) and multimodal (F8 to F13) optimization test functions introduced in the preceding subsections. 50 search agents (candidate solutions) are utilized to solve these benchmark optimization problems over 2000 iterations. Finally, the results are illustrated in Tables 14 and 15 for unimodal and multimodal test functions, respectively.

As results are shown in Table 14, the ChOA outperforms all the other algorithms on five of the unimodal optimization benchmark functions (F1, F3, F4, F6, and F7). Besides, Table 15 indicates that this algorithm provides the best results on five of the six multimodal optimization benchmark functions (F8, F9, F10, F12, and F13). For the rest of the unimodal and multimodal optimization benchmark functions, the ChOA is ranked as the second-best after LGWO (F2 and F5), and GWO (F11). Poor performances of the majority of algorithms in Tables 14 and 15 show that such high-dimensional optimization benchmark functions can be very challenging. These results highly evidence that the ChOA algorithm can be very effective for solving high-dimensional optimization problems as well.

To sum up, the results of this subsection indicates that ChOAs propose high exploitation and exploration. First, the proposed individual intelligence (autonomous group in initial iteration) and sexual motivation (chaotic behaviour in final iteration) of chimps in their group hunting promote exploration, enhance the ChOA algorithm to avoid local optima stagnation when solving high-dimensional optimization problems. Secondly, the decreasing shape of **f** for each independent group of chimps, emphasizes exploitation as iteration increases, which results in a very precise estimation of the global optimum.

Table 17
The results of ChOA in comparison with benchmark algorithms for real-world problems.

Problem		Optimization Algorithms							
ID	metric	GA	GSA	PSO	BH	CS	GWO	LGWO	ChOA
RC01	Ave	2.18E + 02	3.12E + 02	2.23E + 02	3.36E + 02	4.02E + 02	2.12E + 02	2.01E + 02	**1.92E + 2**
	STD	5.34E−02	2.18E−02	2.81E−02	3.21E−02	3.18E−02	2.85E-02	1.67E-03	**1.45E-03**
RC04	Ave	−1.38E-01	−3.86E-01	−1.15E-01	−3.60E-00	−3.86E-00	−3.52E-01	−3.56E-01	**−3.87E-01**
	STD	7.98E−01	1.92E−01	2.84E−01	3.41E−01	3.80E−01	3.96E + 00	2.57E + 00	**1.02E−01**
RC11	Ave	12.63E + 01	11.93E + 01	11.62E + 01	10.56E + 01	10.13E + 01	10.17E + 01	10.74E + 01	**9.99E + 01**
	STD	2.76E + 01	1.67E + 01	2.21E + 01	1.79E + 01	1.97E + 01	1.54E + 01	2.34E + 01	**1.34E + 01**
RC14	Ave	9.45E + 04	7.24E + 04	9.82E + 04	9.24E + 04	9.23E + 04	6.12E + 04	**5.89E + 04**	5.92E + 04
	STD	5.84E + 00	5.62E + 02	1.91E + 00	9.68E + 02	8.10E + 00	1.87E + 01	9.59E + 00	**1.01E + 00**
RC16	Ave	4.59E-02	4.09E-02	4.52E-02	4.58E-02	4.89E-02	4.09E-02	4.98E-02	**3.99E-02**
	STD	2.41E−04	8.58E-02	5.32E−03	2.51E−02	6.98E−03	8.58E-02	3.71E-01	**1.58E-02**
RC23	Ave	3.18E + 01	3.75E + 01	2.88E + 01	2.85E + 01	2.27E + 01	2.82E + 01	1.98E + 01	2.02E + 01
	STD	6.78E + 00	4.73E + 00	4.21E + 00	6.73E + 00	1.21E + 00	6.27E + 00	**1.02E-02**	1.27E + 00
RC35	Ave	9.78E-02	9.56E-02	9.36E-02	9.74E-02	9.56E-02	9.24E-02	9.81E-02	**9.01E-02**
	STD	8.64E-02	9.47E-02	9.42E-01	9.64E-01	4.73E+00	6.56E-01	6.15E-01	**5.42E-01**
RC37	Ave	2.85E-02	3.11E-02	3.24E-02	2.94E-02	2.45E-02	3.11E-02	2.92E-02	**2.20E-02**
	STD	1.64E-02	1.24E-02	1.79E-02	2.02E-02	1.40E-02	1.78E-02	1.68E-02	**1.11E-02**
RC45	Ave	4.38E-02	3.92E-02	4.49E-02	4.75E-02	4.12E-02	4.02E-02	**3.89E-02**	3.99E-02
	STD	1.36E-03	1.07E-03	1.54E-03	1.50E-03	1.81E-02	1.47E-03	**1.03E-03**	1.53E-03
RC51	Ave	4.59E + 03	4.91E + 03	5.24E + 03	5.29E + 03	4.89E + 03	4.62E + 03	4.59E + 03	**4.55E + 03**
	STD	1.49E + 00	2.94E + 00	2.84E + 00	2.75E + 00	2.21E + 00	2.83E + 00	1.90E + 00	**1.09E + 00**

3.4. Results and analysis of real-world problems

In this section, the effectiveness of ChOA is investigated using Ten real-world problems from IEEE CEC2020 (Kumar et al., 2019). It's worth mentioning that these evaluations are carried out according to the guidelines of CEC2020. Note that the complete description of these real-world problems is described in CEC2020 (Kumar et al., 2019). However, the general description of the real-world problems, used in this section, can be obtained from Table 16. Also, the results of these evaluations are shown in Table 17.

Based on Table 17, it is seen that the conventional LGWO represents the best performance in two cases Optimal Design of Industrial refrigeration System (RC23) and SOPWM for 3-level Inverters (RC45). The proposed ChOA provides the best results in the remaining real-world optimization test cases (RC01, RC04, RC11, RC14, RC16, RC35, RC37, and RC51). Therefore, In comparison with GA, GSA, PSO, BH, CS, GWO and LGWO algorithms, the ChOA's statistical results indicate that it can be considered as the best optimization algorithm in working with real-world optimization test problems. The LGWO is also the second best algorithm in dealing with these test cases.

4. Conclusion

This paper proposed a novel hunting-based optimization algorithm called ChOA. The proposed ChOA mimicked the social diversity and hunting behaviour of chimps. Four hunting behaviors (driving, chasing, blocking, and attacking), several operators such as diverse intelligence and sexual motivation, and also four kinds of chimps were proposed and mathematically modelled for supplying the ChOA with high exploitation and exploration. The performance of ChOA was benchmarked on 30 mathematical test functions, 13 high-dimensional test problems, and 10 real-world optimization problems in terms of exploration, exploitation, local optima avoidance, and convergence rate. As per the superior results of the ChOA on the majority of the unimodal test functions and convergence curves, it can be concluded that the proposed algorithm benefits from convergence rate and high exploitation. The main reason for the high exploitation and convergence speed is due to the proposed semi-deterministic feature of chaotic maps. High exploration of ChOA can be concluded from the results of multimodal and composite test functions, which is because of dividing chimps into independent groups and allowing them to have different searching behaviour.

Finally, the special decreasing shapes of various **f** parameter evidenced that the ChOA requires chimps to move suddenly in the initial steps of the algorithm and locally in the final steps of the algorithm, which leads to a gentle transition and balance between exploitation and exploration. The ChOA was compared to nine well-known optimization algorithms in the literature: PSO, GA, GSA, BH, GWO, CS, BBO, ALO, and LGWO. Wilcoxon statistical tests were also conducted when comparing other optimization algorithms. The results indicated that the ChOA provides very competitive results and outperforms other optimization algorithms in the majority of benchmark functions. The statistical test also proved that the results were statistically significant for the ChOA. Therefore, it may be concluded from the comparative results that the proposed ChOA is able to be employed as an alternative optimizer for optimizing various high-dimensional optimization problems.

The paper also considered solving ten real-world optimization problems using the ChOA. The results of the ChOA on these real-world problems were compared to a wide range of other optimization algorithms. The comparative results indicated that the ChOA is able to solve real-world optimization problems with unknown search spaces as well. Other conclusion remarks that can be made from the results of this study are as follows:

- Dividing chimps in independent groups guarantees exploration of the search space, particularly for problems of higher dimensionality.
- The proposed semi-deterministic feature of chaotic maps emphasizes the exploitation ability of the ChOA.
- The use of chaotic maps assists the ChOA algorithm to resolve local optima stagnations.
- Local optima avoidance is very high since the ChOA algorithm employs a four kind of population of search agents to approximate the global optimum.
- The special decreasing shapes of various **f** parameter promotes exploitation and convergence rate as the iteration counter increases.
- Chimps memorize search space information over the course of iteration.
- ChOA almost uses memory to keep the best solution acquired so far.

- ChOA generally has a few parameters to adjust.
- Considering the parallel structure of independent groups and the simplicity of ChOA, it is very easy to implement the proposed algorithm.
- Chimps are not quite similar in terms of ability and intelligence, but they all perform their tasks as members of a hunting group. So, each individual's ability can be useful in a special phase of the hunting event.

Several research directions can be recommended for future studies with the proposed algorithm. Utilizing the ChOA to tackle different optimization problems in different industrial tasks. Also, modifying ChOA to solve multi- and many-objective optimization problems can be investigated as a good contribution. Besides, the effectiveness of ChOA can be compared with other hunting-based optimizers for solving different optimization problems. Another research direction is to investigate the effectiveness of other chaotic maps in improving the performance of the ChOA algorithm. Finally, it is possible to design a discrete extension of ChOA.

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