Sea-Horse Optimizer

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Sea-Horse Optimizer (SHO)

- Fue desarrollada por Shijie Zhao et. al. en el año 2022 [a].
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoreamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

[a] *Sea-horse optimizer: a novel nature-inspired meta-heuristic for global optimization problems*, Springer Link (2022)

SHO: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$\sigma = \frac{\Gamma(1+\lambda)\cdot \sin\left(\frac{\pi\lambda}{2}\right)}{\Gamma\left(\frac{1+\lambda}{2}\right)\cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}} \quad (1)$$

$$\text{Levy}(z) = s \cdot \frac{W \cdot \sigma}{|k|^{1/\lambda}} \quad (2)$$

- Donde:
  - r_1 es un número aleatorio entre $0, 1$
    - S representa la locación de la presa
    - d_i representa la distancia desde el badger y la presa
    - r_2 es un número aleatorio entre $0, 1$

SHO: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$\rho = u \cdot exp(\theta v) \tag{3}$$

$$x = \rho \cdot \cos(\theta) \tag{4}$$

$$y = \rho \cdot \sin(\theta) \tag{5}$$

$$z = \rho \cdot \theta \tag{6}$$

- Donde:
  - en α
    - C es una constante ≥ 1, por defecto $= 2$
  - $x_p rey$ es la posición de la presa, que es la mejor posición encontrada hasta el momento
  - r_3 ,r_4 ,r_5 ,r_6 son números aleatorio entre $0, 1$
  - $eq.(6)$ representa la fase de excavación del badger

- Ecuaciones de movimientos general

$$X_{1_{new}}(t+1) = X_i(t) + \text{Levy}(\lambda)((\text{Best}(t) - X_i(t)) \cdot x \cdot y \cdot z + \text{Best}(t)) \quad (7)$$

- Donde:
  - $eq8$ representa cuando el badger va hacia la colmena
  - r_7 es un número aleatorio entre $0, 1$

- Ecuaciones de movimientos general

$$X_{1_{\text{new}}}(t+1) = X_i(t) + \text{rand} \cdot l \cdot \beta_t \cdot (X_i(t) - \beta_t \cdot \text{Best}(t)) \quad (8)$$

- Donde:
  - $eq8$ representa cuando el badger va hacia la colmena
  - r_7 es un número aleatorio entre $0, 1$

SHO: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$\alpha = \left(1 - \frac{t}{T}\right)^{\frac{2t}{T}} \tag{9}$$

$$X_{\text{new}}^{2}(t+1) = \begin{cases} \alpha \cdot (\text{Best} - \text{rand} \cdot X_{\text{new}}^{1}(t)) + (1-\alpha) \cdot \text{Best}, r_2 > 0{,}1 \\ (1-\alpha) \cdot (X_{\text{new}}^{1}(t) - \text{rand} \cdot \text{Best}) + \alpha \cdot X_{\text{new}}^{1}(t), r_2 \leq 0{,}1 \end{cases} \tag{10}$$

- Donde:
  - $eq8$ representa cuando el badger va hacia la colmena
  - r_7 es un número aleatorio entre $0,1$

- Ecuaciones de movimientos general

$$X_i^{\text{father}} = X_{\text{sort}}^2(1 : N/2) \tag{11}$$

$$X_i^{\text{mother}} = X_{\text{sort}}^2 \, (N/2 + 1 : N) \tag{12}$$

$$X_i^{\text{offspring}} = r_3 \cdot X_i^{\text{father}} + (1 - r_3) \cdot X_i^{\text{mother}} \tag{13}$$

- Donde:
  - $eq8$ representa cuando el badger va hacia la colmena
  - r_7 es un número aleatorio entre $0, 1$

SHO: Pseudocódigo

Algorithm 1 Sea-Horse Optimizer

```
Input: Population X={x_1, x_2, ..., x_N}
Output: Updated population X'={x'_1, x'_2, ..., x'_n} and Best
Initialize random sea-horse population X; constants u, v, l, λ and s
for it = 1 to MaxIt do
    calulate the fitness of each X
    select Best
    β = randn(N, dim)
    r1 = randn(dim)
    for i=0 to N do
        for j=0 to dim do
            if r1[j] > 0
                w = random()
                k = random()
                σ = (Γ(1 + λ) ∗ sin(π ∗ λ/2)) / (Γ((1 + λ)/2) ∗ λ ∗ 2^((λ − 1)/2)
                levy = levy() = s * ((w ∗ σ) / (|k|^(1/λ)))
                θ = random(0,2*π)
                p = u * exp(θ * v)
                x = p * cos(θ)
                y = p * sin(θ)
                z = p * θ
                X[i,j]=(X[i,j] + levy*(Best[j]-X[i,j])*x*y*z + Best[j])
            else
                X[i,j] = X[i,j] + random()*l*β[i,j]*(X[i,j]-β[i,j]*Best[j])
        Handle variables out of bound
    α = (1-it/maxIter)^((2*it)/maxIter)
```

SHO: Pseudocódigo

```
        for i=0 to N do
            for j=0 to dim do
                r2 = random()
                if r2 > 0.1
                    X[i,j] = α * (Best[j] - random()*X[i,j])
                else
                    X[i,j] = ((1-α) * (X[i,j] - random() * Best[j]) + α * X[i,j])
            Handle variables out of bound. Calculate the fitness value of each sea-horse
        sortIndex = argsort(fitness)
        father = X[sortIndex[:N // 2]]
        mother = X[sortIndex[N // 2:]]
        for k=0 to N//2 do
            r3 = random()
            for j=0 to dim do
                offspring[k,j] = r3 * father[k,j] + (1 - r3) * mother[k,j]
            Handle variables out of bound. Calculate the fitness value of each offspring
        Select the next iteration population from the offspring and X ranked top N in fitness values
    return updated population X' and Best
```

Considerando

$$\text{Min } z = x_1^2 + x_2^2$$

Sujeto a

$$x_1, x_2 \in [-100, 100]$$

Configuración inicial de SHO:

- Tamaño de la población: 2 individuos
- Número máximo de iteraciones: 100 iteraciones
- $u, v, l = 0{,}05$
- $\lambda = 1{,}5$
- $s = 0{,}01$

Soluciones inciales:
ind 1: [-84.0678 , 92.4701] / fitness: 15618.1074
ind 2: [-2.6283 , 78.794] / fitness: 6215.3974

Mejor solución:
ind 1: [-2.6283 , 78.794] / fitness: 6215.3974

Ecuaciones generales de la iteración 1:

$\beta = randn(N, dim)$

$r_1 = randn(dim)$

Ecuación general SHO:

$$\beta = randn(2,2) = \begin{bmatrix} 0{,}7529 & 0{,}8498 \\ 0{,}3451 & -1{,}5823 \end{bmatrix}$$

$$r_1 = randn(2) = \begin{bmatrix} 0{,}2527 & -1{,}1805 \end{bmatrix}$$

$$r_1{}^1 > 0 \rightarrow 0{,}2527 > 0$$
$$w = random() = 0{,}7368$$
$$k = random() = 0{,}5470$$

$$\sigma = \frac{\Gamma(\lambda+1) \cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2}) \cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s \cdot \frac{w \cdot \sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293 \cdot -1{,}0000)}{0{,}9064 \cdot 1{,}5 \cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01 \cdot \frac{0{,}7368 \cdot -0{,}8222}{|0{,}5470^{\frac{1}{1{,}5}}|} = -0{,}0091$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 1

$r_1{}^1 > 0 \rightarrow 0{,}2527 > 0$
$\theta = random(0, 2\pi) = 5{,}6548$
$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(5{,}6548 \cdot 0{,}05) = 0{,}0663$
$x = \rho \cdot cos(\theta) = 0{,}0663 \cdot cos(5{,}6548) = 0{,}0537$
$y = \rho \cdot sin(\theta) = 0{,}0663 \cdot sin(5{,}6548) = -0{,}0390$
$z = \rho \cdot \theta = 0{,}0663 \cdot 5{,}6548 = 0{,}3751$

$$X_{1,1} = X_{1,1} + levy \cdot (Best_1 - X_{1,1}) \cdot x \cdot y \cdot z + Best_1$$

$X_{1,1} = -84{,}0678 - 0{,}0091 \cdot (-2{,}6283 + 84{,}0678) \cdot 0{,}0537 \cdot -0{,}0390 \cdot$
$0{,}3751 - 2{,}6283 = -86{,}6955$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 1

$${r_1}^2 \leq 0 \rightarrow -1{,}1805 \leq 0$$

$$X_{1,2} = X_{1,2} + random() \cdot l \cdot \beta_{1,2} \cdot (X_{1,2} - \beta_{1,2} \cdot Best_2)$$

$$X_{1,2} = 92{,}4701 + 0{,}3058 \cdot 0{,}05 \cdot 0{,}8498 \cdot (92{,}4701 - 0{,}8498 \cdot 78{,}7940) = 92{,}8016$$

$r_1{}^1 > 0 \rightarrow 0{,}2527 > 0$
$w = random() = 0{,}9294$
$k = random() = 0{,}9164$

$\sigma = \frac{\Gamma(\lambda+1) \cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2}) \cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}}$
$levy = s \cdot \frac{w \cdot \sigma}{|k^{\frac{1}{\lambda}}|}$

$\sigma = \frac{1{,}3293 \cdot -1{,}0000)}{0{,}9064 \cdot 1{,}5 \cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$
$levy = 0{,}01 \cdot \frac{0{,}9294 \cdot -0{,}8222}{|0{,}9164^{\frac{1}{1{,}5}}|} = -0{,}0081$

$$r_1{}^1 > 0 \rightarrow 0{,}2527 > 0$$
$$\theta = random(0, 2\pi) = 4{,}5770$$
$$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(4{,}5770 \cdot 0{,}05) = 0{,}0629$$
$$x = \rho \cdot cos(\theta) = 0{,}0629 \cdot cos(4{,}5770) = -0{,}0085$$
$$y = \rho \cdot sin(\theta) = 0{,}0629 \cdot sin(4{,}5770) = -0{,}0623$$
$$z = \rho \cdot \theta = 0{,}0629 \cdot 4{,}5770 = 0{,}2877$$

$$X_{2,1} = X_{2,1} + levy \cdot (Best_1 - X_{2,1}) \cdot x \cdot y \cdot z + Best_1$$

$$X_{2,1} = -2{,}6283 - 0{,}0081 \cdot (-2{,}6283 + 2{,}6283) \cdot -0{,}0085 \cdot -0{,}0623 \cdot 0{,}2877 - 2{,}6283 = -5{,}2567$$

SHO: Ejemplo prático - ind 2 - dim 2 - iter 1

$r_1{}^2 \leq 0 \rightarrow -1{,}1805 \leq 0$

$X_{2,2} = X_{2,2} + random() \cdot l \cdot \beta_{2,2} \cdot (X_{2,2} - \beta_{2,2} \cdot Best_2)$

$X_{2,2} = 78{,}7940 + 0{,}0482 \cdot 0{,}05 \cdot -1{,}5823 \cdot (78{,}7940 + 1{,}5823 \cdot 78{,}7940) = 78{,}0181$

SHO: Ejemplo prático - iter 1

$$\alpha = \frac{1 - iter}{MaxIter}^{\frac{2 \cdot it}{MaxIter}}$$

$$\alpha = \frac{1 - 1}{100}^{\frac{2 \cdot 1}{100}} = 0{,}9998$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 1

$r_2 = random() = 0{,}4455$
$r_2 > 0{,}1 \rightarrow 0{,}4455 > 0{,}1$

$$X_{1,1} = \alpha \cdot (Best_1 - random() \cdot X_{1,1})$$

$$X_{1,1} = 0{,}9998 \cdot (-2{,}6283 - 0{,}2547 \cdot -86{,}6955) = 19{,}4470$$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 1

$r_2 = random() = 0{,}6132$
$r_2 > 0{,}1 \rightarrow 0{,}6132 > 0{,}1$

$$X_{1,2} = \alpha \cdot (Best_2 - random() \cdot X_{1,2})$$

$$X_{1,2} = 0{,}9998 \cdot (78{,}7940 - 0{,}3671 \cdot 92{,}8016) = 44{,}7143$$

$r_2 = random() = 0{,}0396$
$r_2 \leq 0{,}1 \rightarrow 0{,}0396 \leq 0{,}1$

$$X_{2,1} = (1 - \alpha) \cdot (X_{2,1} - random() \cdot Best_1) + \alpha \cdot X_{2,1}$$

$$X_{2,1} = (1 - 0{,}9998) \cdot (-5{,}2567 - 0{,}8647 \cdot -2{,}6283) + 0{,}9998 \cdot -5{,}2567 = -5{,}2562$$

SHO: Ejemplo prático - ind 2 - dim 2 - iter 1

$r_2 = random() = 0{,}9225$
$r_2 > 0{,}1 \rightarrow 0{,}9225 > 0{,}1$

$$X_{2,2} = \alpha \cdot (Best_2 - random() \cdot X_{2,2})$$

$$X_{2,2} = 0{,}9998 \cdot (78{,}7940 - 0{,}2513 \cdot 78{,}0181) = 59{,}1740$$

SHO: Ejemplo prático - iter 1

$f(X) = \begin{bmatrix} 2377{,}5537 & 3529{,}195 \end{bmatrix}$
$SortFitness = \begin{bmatrix} 2377{,}5537 & 3529{,}195 \end{bmatrix}$

$father = SortedFitness[: N//2] = SortedFitness[: 2//2] =$
$\begin{bmatrix} 19{,}447 & 44{,}7143 \end{bmatrix}$
$mother = SortedFitness[N//2 :] = SortedFitness[2//2 :] =$
$\begin{bmatrix} -5{,}2562 & 59{,}174 \end{bmatrix}$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 1

$N//2 = 1$
$r_3 = random() = 0{,}0892$

$$offspring_{1,1} = r_3 \cdot father_{1,1} + (1 - r_3) \cdot mother_{1,1}$$

$$offspring_{1,1} = 0{,}0892 \cdot 19{,}4470 + (1 - 0{,}0892) \cdot -5{,}2562 = -3{,}0535$$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 1

$N//2 = 1$
$r_3 = random() = 0{,}0892$

$offspring_{1,2} = r_3 \cdot father_{1,2} + (1 - r_3) \cdot mother_{1,2}$

$offspring_{1,2} = 0{,}0892 \cdot 44{,}7143 + (1 - 0{,}0892) \cdot 59{,}1740 = 57{,}8847$

SHO: Ejemplo prático - iter 1

$$X = \begin{bmatrix} 19{,}447 & 44{,}7143 \\ -5{,}2562 & 59{,}174 \end{bmatrix}$$
$$f(X) = \begin{bmatrix} 2377{,}5537 & 3529{,}195 \end{bmatrix}$$
$$offspring = \begin{bmatrix} -3{,}0535 & 57{,}8847 \end{bmatrix}$$
$$f(offspring) = 3359{,}966$$

$$sort = sort(concatenate(f(X), f(offspring))) =$$
$$\begin{bmatrix} 2377{,}5537 & 3359{,}966 & 3529{,}195 \end{bmatrix}$$
$$sort[:N] = sort[:2] = \begin{bmatrix} 2377{,}5537 & 3359{,}966 \end{bmatrix}$$

$$X = \begin{bmatrix} 19{,}447 & 44{,}7143 \\ -5{,}2562 & 59{,}174 \end{bmatrix}$$

SHO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 1:
ind 1: [19.447 44.7143], infactibles: 0
ind 2: [-3.0535 57.8847], infactibles: 0

Reparación de soluciones:
ind 1: [19.447 44.7143] / fitness: 2377.5537
ind 2: [-3.0535 57.8847] / fitness: 3359.9660

Mejor solución:
ind 1: [19.447 44.7143] / fitness: 2377.5537

Ecuaciones generales de la iteración 2:
$\beta = randn(N, dim)$
$r_1 = randn(dim)$

Ecuación general SHO:

$$\beta = randn(2,2) = \begin{bmatrix} -1{,}3726 & 0{,}7389 \\ 1{,}1418 & -0{,}4538 \end{bmatrix}$$
$$r_1 = randn(2) = \begin{bmatrix} 0{,}3056 & 0{,}3204 \end{bmatrix}$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 2

$r_1{}^1 > 0 \rightarrow 0{,}3056 > 0$
$w = random() = 0{,}4739$
$k = random() = 0{,}9194$

$\sigma = \frac{\Gamma(\lambda+1) \cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2}) \cdot \lambda \cdot 2^{\frac{\lambda-1}{2}}}$
$levy = s \cdot \frac{w \cdot \sigma}{|k^{\frac{1}{\lambda}}|}$

$\sigma = \frac{1{,}3293 \cdot -1{,}0000)}{0{,}9064 \cdot 1{,}5 \cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$
$levy = 0{,}01 \cdot \frac{0{,}4739 \cdot -0{,}8222}{|0{,}9194^{\frac{1}{1{,}5}}|} = -0{,}0041$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 2

$$r_1{}^1 > 0 \rightarrow 0{,}3056 > 0$$
$$\theta = random(0, 2\pi) = 0{,}5447$$
$$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(0{,}5447 \cdot 0{,}05) = 0{,}0514$$
$$x = \rho \cdot cos(\theta) = 0{,}0514 \cdot cos(0{,}5447) = 0{,}0439$$
$$y = \rho \cdot sin(\theta) = 0{,}0514 \cdot sin(0{,}5447) = 0{,}0266$$
$$z = \rho \cdot \theta = 0{,}0514 \cdot 0{,}5447 = 0{,}0280$$

$$X_{1,1} = X_{1,1} + levy \cdot (Best_1 - X_{1,1}) \cdot x \cdot y \cdot z + Best_1$$

$$X_{1,1} = 19{,}4470 - 0{,}0041 \cdot (19{,}4470 - 19{,}4470) \cdot 0{,}0439 \cdot 0{,}0266 \cdot 0{,}0280 + 19{,}4470 = 38{,}8939$$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 2

$$r_1{}^2 > 0 \rightarrow 0{,}3204 > 0$$
$$w = random() = 0{,}2684$$
$$k = random() = 0{,}6011$$

$$\sigma = \frac{\Gamma(\lambda+1)\cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2})\cdot\lambda\cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s \cdot \frac{w\cdot\sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293\cdot -1{,}0000)}{0{,}9064\cdot 1{,}5\cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01 \cdot \frac{0{,}2684\cdot -0{,}8222}{|0{,}6011^{\frac{1}{1{,}5}}|} = -0{,}0031$$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 2

$r_1{}^2 > 0 \rightarrow 0{,}3204 > 0$
$\theta = random(0, 2\pi) = 5{,}7941$
$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(5{,}7941 \cdot 0{,}05) = 0{,}0668$
$x = \rho \cdot cos(\theta) = 0{,}0668 \cdot cos(5{,}7941) = 0{,}0590$
$y = \rho \cdot sin(\theta) = 0{,}0668 \cdot sin(5{,}7941) = -0{,}0314$
$z = \rho \cdot \theta = 0{,}0668 \cdot 5{,}7941 = 0{,}3871$

$X_{1,2} = X_{1,2} + levy \cdot (Best_2 - X_{1,2}) \cdot x \cdot y \cdot z + Best_2$

$X_{1,2} = 44{,}7143 - 0{,}0031 \cdot (44{,}7143 - 44{,}7143) \cdot 0{,}0590 \cdot -0{,}0314 \cdot 0{,}3871 + 44{,}7143 = 89{,}4286$

SHO: Ejemplo prático - ind 2 - dim 1 - iter 2

$$r_1{}^1 > 0 \rightarrow 0{,}3056 > 0$$
$$w = random() = 0{,}0051$$
$$k = random() = 0{,}8530$$

$$\sigma = \frac{\Gamma(\lambda+1)\cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2})\cdot\lambda\cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s \cdot \frac{w\cdot\sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293\cdot -1{,}0000)}{0{,}9064\cdot 1{,}5\cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01 \cdot \frac{0{,}0051\cdot -0{,}8222}{|0{,}8530^{\frac{1}{1{,}5}}|} = -0{,}0000$$

SHO: Ejemplo prático - ind 2 - dim 1 - iter 2

$r_1{}^1 > 0 \rightarrow 0{,}3056 > 0$
$\theta = random(0, 2\pi) = 1{,}2299$
$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(1{,}2299 \cdot 0{,}05) = 0{,}0532$
$x = \rho \cdot cos(\theta) = 0{,}0532 \cdot cos(1{,}2299) = 0{,}0178$
$y = \rho \cdot sin(\theta) = 0{,}0532 \cdot sin(1{,}2299) = 0{,}0501$
$z = \rho \cdot \theta = 0{,}0532 \cdot 1{,}2299 = 0{,}0654$

$$X_{2,1} = X_{2,1} + levy \cdot (Best_1 - X_{2,1}) \cdot x \cdot y \cdot z + Best_1$$

$$X_{2,1} = -3{,}0535 - 0{,}0000 \cdot (19{,}4470 + 3{,}0535) \cdot 0{,}0178 \cdot 0{,}0501 \cdot 0{,}0654 + 19{,}4470 = 16{,}3934$$

SHO: Ejemplo prático - ind 2 - dim 2 - iter 2

$$r_1{}^2 > 0 \rightarrow 0{,}3204 > 0$$
$$w = random() = 0{,}5778$$
$$k = random() = 0{,}2544$$

$$\sigma = \frac{\Gamma(\lambda+1)\cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2})\cdot\lambda\cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s \cdot \frac{w\cdot\sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293\cdot -1{,}0000)}{0{,}9064\cdot 1{,}5\cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01 \cdot \frac{0{,}5778\cdot -0{,}8222}{|0{,}2544^{\frac{1}{1{,}5}}|} = -0{,}0118$$

$r_1{}^2 > 0 \rightarrow 0{,}3204 > 0$
$\theta = random(0, 2\pi) = 4{,}5619$
$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(4{,}5619 \cdot 0{,}05) = 0{,}0628$
$x = \rho \cdot cos(\theta) = 0{,}0628 \cdot cos(4{,}5619) = -0{,}0094$
$y = \rho \cdot sin(\theta) = 0{,}0628 \cdot sin(4{,}5619) = -0{,}0621$
$z = \rho \cdot \theta = 0{,}0628 \cdot 4{,}5619 = 0{,}2865$

$$X_{2,2} = X_{2,2} + levy \cdot (Best_2 - X_{2,2}) \cdot x \cdot y \cdot z + Best_2$$

$$X_{2,2} = 57{,}8847 - 0{,}0118 \cdot (44{,}7143 - 57{,}8847) \cdot -0{,}0094 \cdot -0{,}0621 \cdot 0{,}2865 + 44{,}7143 = 102{,}5991$$

SHO: Ejemplo prático - iter 2

$$\alpha = \frac{1 - iter}{MaxIter}^{\frac{2 \cdot it}{MaxIter}}$$

$$\alpha = \frac{1 - 2}{100}^{\frac{2 \cdot 2}{100}} = 0{,}9992$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 2

$r_2 = random() = 0{,}6324$
$r_2 > 0{,}1 \rightarrow 0{,}6324 > 0{,}1$

$X_{1,1} = \alpha \cdot (Best_1 - random() \cdot X_{1,1})$

$X_{1,1} = 0{,}9992 \cdot (19{,}4470 - 0{,}1089 \cdot 38{,}8939) = 15{,}1993$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 2

$r_2 = random() = 0{,}8826$
$r_2 > 0{,}1 \rightarrow 0{,}8826 > 0{,}1$

$$X_{1,2} = \alpha \cdot (Best_2 - random() \cdot X_{1,2})$$

$$X_{1,2} = 0{,}9992 \cdot (44{,}7143 - 0{,}0022 \cdot 89{,}4286) = 44{,}4785$$

$r_2 = random() = 0{,}5063$
$r_2 > 0{,}1 \rightarrow 0{,}5063 > 0{,}1$

$$X_{2,1} = \alpha \cdot (Best_1 - random() \cdot X_{2,1})$$

$$X_{2,1} = 0{,}9992 \cdot (19{,}4470 - 0{,}9708 \cdot 16{,}3934) = 3{,}5291$$

SHO: Ejemplo prático - ind 2 - dim 2 - iter 2

$$r_2 = random() = 0{,}4876$$
$$r_2 > 0{,}1 \rightarrow 0{,}4876 > 0{,}1$$

$$X_{2,2} = \alpha \cdot (Best_2 - random() \cdot X_{2,2})$$

$$X_{2,2} = 0{,}9992 \cdot (44{,}7143 - 0{,}7514 \cdot 102{,}5991) = -32{,}3527$$

SHO: Ejemplo prático - iter 2

$$f(X) = \begin{bmatrix} 2209{,}3531 & 1059{,}153 \end{bmatrix}$$
$$SortFitness = \begin{bmatrix} 1059{,}153 & 2209{,}3531 \end{bmatrix}$$

$$father = SortedFitness[: N//2] = SortedFitness[: 2//2] = \begin{bmatrix} 3{,}5291 & -32{,}3527 \end{bmatrix}$$
$$mother = SortedFitness[N//2 :] = SortedFitness[2//2 :] = \begin{bmatrix} 15{,}1993 & 44{,}4785 \end{bmatrix}$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 2

$N//2 = 1$
$r_3 = random() = 0{,}9356$

$$offspring_{1,1} = r_3 \cdot father_{1,1} + (1 - r_3) \cdot mother_{1,1}$$

$$offspring_{1,1} = 0{,}9356 \cdot 3{,}5291 + (1 - 0{,}9356) \cdot 15{,}1993 = 4{,}2801$$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 1

$N//2 = 1$
$r_3 = random() = 0{,}9356$

$offspring_{1,2} = r_3 \cdot father_{1,2} + (1 - r_3) \cdot mother_{1,2}$

$offspring_{1,2} = 0{,}9356 \cdot -32{,}3527 + (1 - 0{,}9356) \cdot 44{,}4785 = -27{,}4086$

SHO: Ejemplo prático - iter 2

$$X = \begin{bmatrix} 15{,}1993 & 44{,}4785 \\ 3{,}5291 & -32{,}3527 \end{bmatrix}$$
$$f(X) = \begin{bmatrix} 2209{,}3531 & 1059{,}153 \end{bmatrix}$$
$$offspring = \begin{bmatrix} 4{,}2801 & -27{,}4086 \end{bmatrix}$$
$$f(offspring) = 769{,}5518$$

$$sort = sort(concatenate(f(X), f(offspring))) =$$
$$\begin{bmatrix} 769{,}5518 & 1059{,}153 & 2209{,}3531 \end{bmatrix}$$
$$sort[:N] = sort[:2] = \begin{bmatrix} 769{,}5518 & 1059{,}153 \end{bmatrix}$$

$$X = \begin{bmatrix} 15{,}1993 & 44{,}4785 \\ 3{,}5291 & -32{,}3527 \end{bmatrix}$$

SHO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 2:
ind 1: [4.2801 -27.4086], infactibles: 0
ind 2: [3.5291 -32.3527], infactibles: 0

Reparación de soluciones:
ind 1: [4.2801 -27.4086] / fitness: 769.5518
ind 2: [3.5291 -32.3527] / fitness: 1059.1530

Mejor solución:
ind 1: [4.2801 -27.4086] / fitness: 769.5518

SHO: Ejemplo práctico - iter 100

Ecuaciones generales de la iteración 100:

$\beta = randn(N, dim)$

$r_1 = randn(dim)$

Ecuación general SHO:

$$\beta = randn(2,2) = \begin{bmatrix} -0{,}9653 & 0{,}4261 \\ 0{,}2518 & 0{,}1205 \end{bmatrix}$$

$$r_1 = randn(2) = \begin{bmatrix} -0{,}3642 & 1{,}6921 \end{bmatrix}$$

$r_1{}^1 \leq 0 \rightarrow -0{,}3642 \leq 0$

$X_{1,1} = X_{1,1} + random() \cdot l \cdot \beta_{1,1} \cdot (X_{1,1} - \beta_{1,1} \cdot Best_1)$

$X_{1,1} = -0{,}0000 + 0{,}3172 \cdot 0{,}05 \cdot -0{,}9653 \cdot (-0{,}0000 + 0{,}9653 \cdot -0{,}0000) = -0{,}0000$

SHO: Ejemplo prático - ind 1 - dim 2 - iter 100

$r_1{}^2 > 0 \rightarrow 1{,}6921 > 0$
$w = random() = 0{,}8817$
$k = random() = 0{,}3899$

$$\sigma = \frac{\Gamma(\lambda+1)\cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2})\cdot\lambda\cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s\cdot\frac{w\cdot\sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293\cdot -1{,}0000)}{0{,}9064\cdot 1{,}5\cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01\cdot\frac{0{,}8817\cdot -0{,}8222}{|0{,}3899^{\frac{1}{1{,}5}}|} = -0{,}0136$$

$${r_1}^2 > 0 \rightarrow 1{,}6921 > 0$$
$$\theta = random(0, 2\pi) = 0{,}3314$$
$$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(0{,}3314 \cdot 0{,}05) = 0{,}0508$$
$$x = \rho \cdot cos(\theta) = 0{,}0508 \cdot cos(0{,}3314) = 0{,}0481$$
$$y = \rho \cdot sin(\theta) = 0{,}0508 \cdot sin(0{,}3314) = 0{,}0165$$
$$z = \rho \cdot \theta = 0{,}0508 \cdot 0{,}3314 = 0{,}0168$$

$$X_{1,2} = X_{1,2} + levy \cdot (Best_2 - X_{1,2}) \cdot x \cdot y \cdot z + Best_2$$

$$X_{1,2} = -0{,}0000 - 0{,}0136 \cdot (-0{,}0000 + 0{,}0000) \cdot 0{,}0481 \cdot 0{,}0165 \cdot 0{,}0168 - 0{,}0000 = -0{,}0000$$

$r_1{}^1 \leq 0 \rightarrow -0{,}3642 \leq 0$

$X_{2,1} = X_{2,1} + random() \cdot l \cdot \beta_{2,1} \cdot (X_{2,1} - \beta_{2,1} \cdot Best_1)$

$X_{2,1} = -0{,}0000 + 0{,}6838 \cdot 0{,}05 \cdot 0{,}2518 \cdot (-0{,}0000 - 0{,}2518 \cdot -0{,}0000) = -0{,}0000$

$$r_1{}^2 > 0 \rightarrow 1{,}6921 > 0$$
$$w = random() = 0{,}7747$$
$$k = random() = 0{,}4581$$

$$\sigma = \frac{\Gamma(\lambda+1)\cdot sin(\pi\lambda)}{\Gamma(\frac{\lambda+1}{2})\cdot\lambda\cdot 2^{\frac{\lambda-1}{2}}}$$
$$levy = s \cdot \frac{w\cdot\sigma}{|k^{\frac{1}{\lambda}}|}$$

$$\sigma = \frac{1{,}3293\cdot -1{,}0000)}{0{,}9064\cdot 1{,}5\cdot 2^{\frac{1{,}5-1}{2}}} = -0{,}8222$$
$$levy = 0{,}01 \cdot \frac{0{,}7747\cdot -0{,}8222}{|0{,}4581^{\frac{1}{1{,}5}}|} = -0{,}0107$$

$$r_1{}^2 > 0 \rightarrow 1{,}6921 > 0$$
$$\theta = random(0, 2\pi) = 1{,}3534$$
$$\rho = u \cdot exp(\theta v) = 0{,}05 \cdot exp(1{,}3534 \cdot 0{,}05) = 0{,}0535$$
$$x = \rho \cdot cos(\theta) = 0{,}0535 \cdot cos(1{,}3534) = 0{,}0115$$
$$y = \rho \cdot sin(\theta) = 0{,}0535 \cdot sin(1{,}3534) = 0{,}0522$$
$$z = \rho \cdot \theta = 0{,}0535 \cdot 1{,}3534 = 0{,}0724$$

$$X_{2,2} = X_{2,2} + levy \cdot (Best_2 - X_{2,2}) \cdot x \cdot y \cdot z + Best_2$$

$$X_{2,2} = -0{,}0000 - 0{,}0107 \cdot (-0{,}0000 + 0{,}0000) \cdot 0{,}0115 \cdot 0{,}0522 \cdot 0{,}0724 - 0{,}0000 = -0{,}0000$$

SHO: Ejemplo prático - iter 100

$$\alpha = \frac{1-iter}{MaxIter}^{\frac{2\cdot it}{MaxIter}}$$

$$\alpha = \frac{1-100}{100}^{\frac{2\cdot 100}{100}} = 0$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 100

$r_2 = random() = 0{,}0931$
$r_2 \leq 0{,}1 \rightarrow 0{,}0931 \leq 0{,}1$

$$X_{1,1} = (1 - \alpha) \cdot (X_{1,1} - random() \cdot Best_1) + \alpha \cdot X_{1,1}$$

$$X_{1,1} = (1 - 0) \cdot (-0{,}0000 - 0{,}6993 \cdot -0{,}0000) + 0 \cdot -0{,}0000 = -0{,}0000$$

$r_2 = random() = 0{,}6537$
$r_2 \; > 0{,}1 \rightarrow 0{,}6537 > 0{,}1$

$X_{1,2} = \alpha \cdot (Best_2 - random() \cdot X_{1,2})$

$X_{1,2} = 0{,}0000 \cdot (-0{,}0000 - 0{,}8234 \cdot -0{,}0000) = 0{,}0000$

SHO: Ejemplo prático - ind 2 - dim 1 - iter 100

$r_2 = random() = 0{,}8348$
$r_2 \ > 0{,}1 \rightarrow 0{,}8348 > 0{,}1$

$X_{2,1} = \alpha \cdot (Best_1 - random() \cdot X_{2,1})$

$X_{2,1} = 0{,}0000 \cdot (-0{,}0000 - 0{,}5982 \cdot -0{,}0000) = 0{,}0000$

SHO: Ejemplo prático - ind 2 - dim 2 - iter 100

$r_2 = random() = 0{,}4558$
$r_2 \;\; > 0{,}1 \rightarrow 0{,}4558 > 0{,}1$

$$X_{2,2} = \alpha \cdot (Best_2 - random() \cdot X_{2,2})$$

$$X_{2,2} = 0{,}0000 \cdot (-0{,}0000 - 0{,}3720 \cdot -0{,}0000) = 0{,}0000$$

SHO: Ejemplo prático - iter 100

$$f(X) = \begin{bmatrix} 5{,}0237e-203 & 0 \end{bmatrix}$$
$$SortFitness = \begin{bmatrix} 0 & 5{,}0237e-203 \end{bmatrix}$$

$$father = SortedFitness[: N//2] = SortedFitness[: 2//2] = \begin{bmatrix} 0 & 0 \end{bmatrix}$$
$$mother = SortedFitness[N//2 :] = SortedFitness[2//2 :] = \begin{bmatrix} -7{,}0878e-102 & 0 \end{bmatrix}$$

SHO: Ejemplo prático - ind 1 - dim 1 - iter 100

$N//2 = 1$
$r_3 = random() = 0{,}6238$

$offspring_{1,1} = r_3 \cdot father_{1,1} + (1 - r_3) \cdot mother_{1,1}$

$offspring_{1,1} = 0{,}6238 \cdot 0{,}0000 + (1 - 0{,}6238) \cdot -0{,}0000 = -0{,}0000$

$N//2 = 1$
$r_3 = random() = 0{,}6238$

$$offspring_{1,2} = r_3 \cdot father_{1,2} + (1 - r_3) \cdot mother_{1,2}$$

$$offspring_{1,2} = 0{,}6238 \cdot 0{,}0000 + (1 - 0{,}6238) \cdot 0{,}0000 = 0{,}0000$$

SHO: Ejemplo prático - iter 100

$$X = \begin{bmatrix} -7{,}0878e-102 & 0 \\ 0 & 0 \end{bmatrix}$$
$$f(X) = \begin{bmatrix} 5{,}0237e-203 & 0 \end{bmatrix}$$
$$offspring = \begin{bmatrix} -2{,}6662e-102 & 0 \end{bmatrix}$$
$$f(offspring) = 7{,}1084e-204$$

$$sort = sort(concatenate(f(X), f(offspring))) =$$
$$\begin{bmatrix} 0 & 7{,}1084e-204 & 5{,}0237e-203 \end{bmatrix}$$
$$sort[:N] = sort[:2] = \begin{bmatrix} 0 & 7{,}1084e-204 \end{bmatrix}$$

$$X = \begin{bmatrix} -7{,}0878e-102 & 0 \\ 0 & 0 \end{bmatrix}$$

SHO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 100:
ind 1: [0. 0.], infactibles: 0 ind 2: [-2.6662e-102 0.0000e+000], infactibles: 0

Reparación de soluciones:
ind 1: [0. 0.] / fitness: 0.0000
ind 2: [-2.6662e-102 0.0000e+000] / fitness: 0.0000

Mejor solución:
ind 1: [0. 0.] / fitness: 0.0000