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A new metaheuristic approach based on agent systems principles

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ABSTRACT

Agent-based modeling is a relatively new approach to model complex systems composed of agents whose behavior is described using simple rules. As a consequence of the agent interactions emerges a complex global behavioral pattern not explicitly programmed. In the last decade, an increasing number of metaheuristic techniques have been reported in the literature where authors claim their novelty and their abilities to perform as powerful optimization methods. Although these schemes emulate very different processes or systems, the rules used to model individual behavior are very similar. The idea behind the design of many metaheuristic methods is to configure a recycled set of rules that has demonstrated to be successful in previous approaches for producing new optimization schemes. Such common rules have been designed without considering the final global result obtained by the individual interactions. On the other hand, agent-based systems provide a solid theory and a set of consistent models that allow characterizing global behavioral patterns produced by the collective interaction of the individuals from a set of simple rules. Under this perspective, several agent-based concepts and models that generate very complex global search behaviors can be used to produce or improve efficient optimization algorithms. In this paper, a new metaheuristic algorithm based on agent systems principles is presented. The proposed method is based on the agent-based model known as “Heroes and Cowards”. This model involves a small set of rules to produce two emergent global patterns that can be considered in terms of the metaheuristic literature as exploration and exploitation stages. To evaluate its performance, the proposed algorithm has been tested in a set of representative benchmark functions, including multimodal, unimodal, and hybrid benchmark formulations. The competitive results demonstrate the promising association between both paradigms.

1. Introduction

Since real-world processes become more interconnected, simple models are no longer enough to analyze them. The wide availability of fast computing resources has allowed the construction and analysis of more complex models. Under such conditions, it has emerged a new field of knowledge known as complex systems [1]. In complex systems, it is studied how systems affect individual behaviors, especially when such individuals have the capacity to influence these systems. In these systems, complex behaviors of higher-level organizations appear as a consequence of the collective interaction of individuals that participate in a self-organizing process [2].

Agent-based modeling [3] represents a new paradigm in artificial intelligence to model complex systems using agents or elements. Agents maintain behaviors that are described by simple rules and are influenced by the collective interaction with other agents. Under this paradigm, global behavioral patterns that have not been directly programmed emerge through the collective interaction among agents. Agent-based models attempt to relate how global regularities may emerge through processes of collective cooperation. Under this scheme, a population of agents maintains a behavior characterized by a set of simple rules. The objective of such rules is to emulate the individual movements of real actors when they interact with their local environment. Although the system is modeled from the individual point of view, its main properties are visualized from a global perspective. The powerful modeling characteristics of the Agent-based models have motivated their use in several applications, which include the prediction of the spread in epidemics [4], the behavior in supply chains [5] and the stock market [6], the characterization of the immune system [7], the understanding about the fall of ancient civilizations [8], the consumer purchasing behavior [9],

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Fire spreading [1], segregation phenomena [10,11] to name a few.

Under the agent-based methodology, several interesting basic global patterns have been proposed to simulate complex phenomena such as diffusion, concentration and insolating, fire spreading, segregation and others. These behavioral patterns have been analyzed in terms of the simple rules that provoke them. In the complex system community, there is a model known as "Heroes and Cowards" [1,12–14] used to illustrate how simple rules can produce complex collective behaviors that are very difficult to reproduce by employing classical modeling techniques. The model produces complex global patterns of concentration and distribution through the interaction of agents that follows simple behavioral rules. In Heroes and Cowards, each agent selects another agent as its "friend" and another as its "enemy". The model consists of two phases. In the first stage, every agent behaves as "coward". Under this condition, the agent moves so that the friend is always located between it and its enemy (in a cowardly manner, hiding from its enemy behind its friend). During this phase, agents are distributed within the space as a consequence of the scape process from the enemy. In the second phase, each agent behaves as "hero". Therefore, the agent presents a strategy where it is moved in a position between its friend and enemy (protecting the friend from the enemy in a heroically). During this stage, agents concentrate around positions marked by the agent distributions.

On the other hand, metaheuristic schemes are abstract optimization models that emulate several biological or social systems. They have been used successfully to solve several complex formulations where traditional optimization schemes lead to poor results. Under these methods, individuals are initially generated by using random values where each individual represents a possible solution actually. An objective function evaluates the quality of each individual in terms of its solution. With this information, at each step, individuals are modified according to some rules that define their behavior within the search space. Such rules correspond to abstract models supposedly extracted from natural or social mechanisms [15]. This process is repeated until a stop criterium has been reached. Metaheuristic methods present two important characteristics that distinguish them from other search methods [16]. First, in contrast to other optimization methods (which consider a single solution), metaheuristic algorithms involve a population of candidate solutions. Second, in metaheuristic methods, there is collective cooperation among the individuals of the population, which promotes characteristics such as adaptation and self-organization. In the last years, several optimization algorithms based on metaheuristic principles have been proposed in the literature. Some of the most popular metaheuristic schemes involve Genetic Algorithm (GA) [17], Artificial Bee Colony (ABC) [18], Differential Evolution (DE) [19], Differential Search (DS) [20], Multi-Verse Optimizer (MVO) [21], the Sine Cosine Algorithm (SCA) [22] and Self-Adaptative Differential Evolution (JADE) [23], Covariance Matrix Adaptation Evolution Strategies (CMA-ES) [24] and Moth-Flame Optimization (MFO) [25].

Agent-based approaches involve a set of autonomous agents that interact with an environment. Under these schemes, all their agents maintain a collective goal. In the system, the sum of the actions from all agents allows reaching the global goal. The main characteristic of agent-based algorithms is the continuous interaction among agents and between them and the environment [26]. This interaction process defines the results produced by the agent system. Metaheuristic methods maintain several properties similar to agent-based systems such as self-organization, the population of individuals and adaptation [27]. In metaheuristic approaches, the elements operate collectively (not individually) to reach a common global objective. The environment with which individuals interact is represented by an objective function and its constraints. According to the literature, there are four ways [28] in which the agent-based systems and metaheuristic schemes are related: (A) assigning to each agent one subtask of the optimization problem to solve; (B) assigning to each agent a candidate solution; (C) assigning to each agent a process of the metaheuristic scheme and (D) assigning a complete metaheuristic scheme to each agent.

(A) One of the associations between metaheuristic methods and agent-based systems is when the optimization task is divided into different subtasks. In these approaches, each agent is assigned to each subtask so that each agent is responsible for solving the subtask under its local perspective. In the end, the global solution is the result of the integration of all partial outcomes. Some examples of these methods include the algorithm introduced in [29], where the decision variables are divided into groups and each variable group is associated with an agent. Then, each agent searches for the optimal value considering only the group of variables designed for it. Another interesting approach is presented in [30,31]. In this approach, it is decomposed the problem into a set of subcomponents. Each subcomponent is evolved by an agent. Then, the Differential Evolution (DE) is used to obtain the optimal value from the results obtained for all agents.

(B) The most representative association between metaheuristic methods and agent-based system is when an agent symbolizes a candidate solution. Therefore, the population of elements can be assumed as a population of agents [32]. From this perspective, an agent not only contains a candidate solution but also is susceptible to exchange information or cooperate with the other agents through specific operators defined by each metaheuristic method [33]. Each individual or agent aims to extend its possibilities for finding the optimal solution of an optimization problem by competing or cooperating collectively with other agents in its population. Several metaheuristic operators can be applied to an agent in order to achieve cooperation, competition, or self-organization [34]. Cooperation is promoted basically through an information exchange scheme such as the crossover operator or an attraction mechanism. For the competition, it is generally used a kind of selection process to identify if an agent or individual presents a better quality than others in terms of its capacity to solve the optimization problem. Self-organization can be produced through metaheuristic operations that induce the movements among agents [35]. Such displacements can be explorative such long movements (a reset mechanism or strong perturbations) or exploitative such a local search. It is important to remark that there is no strict rule to use a specific operation to generate new solutions (or agents) in metaheuristics. In general, it is possible to use any computational strategy to produce new solutions that, considering the information provided by the existent solutions, increase the probability of detecting the global optimal position. The most illustrative examples from this category [27] are several metaheuristic methods such as Particle Swarm Optimization (PSO) algorithm [36], Ant Colony Optimization (ACO) [37], Social Spider Optimization (SSO) [38], Crow Search Algorithm (CSA) [39,40], Gray Wolf Optimizer (GWO) [41], Bat Algorithm (BA) [42] and Cuckoo Search (CS) [43].

(C) Other combination of metaheuristic methods and agent-based systems is when the processes that compose a metaheuristic scheme are considered agents. These processes can be operations or search strategies that integrate the metaheuristic approach [44]. Under this association between metaheuristic and agent-based systems, the operation of the algorithm can be interpreted as the cooperation among different agents whose interactions produce the search strategy [45]. One of the most representative approaches from this category is the evolutionary multi-agent system (EMAS) [46–48]. Under this scheme, evolutionary processes are implemented as agents operate at a population level. The evaluation of the agent performance is conducted through a selection mechanism in order to promote the best-fitted population in terms of the global goal of the system. Other interesting examples of this association are the algorithms reported in [49] and [35].

(D) The last association considers that each metaheuristic method itself represents an autonomous agent. The objective of this approach is to explore the advantages of using agent-based concepts to improve the solutions produced by the involved metaheuristic schemes. Under these methods, each agent (or metaheuristic scheme) performs its particular search process while use cooperatively the solutions produced by other agents in order to refine its result. Therefore, these metaheuristic algorithms (or agents) interact to reach a global goal that represents to identify the global optimal value of the optimization problem. Some examples of these methods involve the approach presented in [50], where several metaheuristic algorithms such as genetic algorithm (GA), iterated local search (ILS) and variable neighborhood search (VNS) are considered agents. Such algorithms exchange information from the search space to refine the global solution. Other similar approaches but with different metaheuristic methods are also reported in [51,52].

The design of a metaheuristic approach is a complex task. Its performance is affected mainly by the interactions among its individuals rather than any other factor [53]. Interactions can be defined as mathematical processes that regulate the kind of contact among individuals and their magnitude. Although all metaheuristic schemes model interactions emulating very different processes or systems, the used operators are very similar [15]. The idea behind the design of many metaheuristic methods is to configure a recycled set of models that have demonstrated to be successful in previous approaches for producing new optimization schemes. Such common mathematical processes have been designed without considering the final global result obtained by the individual interactions. On the other hand, agent-based systems provide a solid theory and a set of consistent models that allow characterizing global behavioral patterns produced by the collective interaction of the individuals from the set of simple rules. Under this perspective, several agent-based concepts and models that generate effective search behaviors can be used to produce or improve efficient optimization algorithms.

In this paper, a new metaheuristic algorithm based on agent systems principles is presented. The paper has two objectives: (I) To demonstrate the efficacy of agent-based models as metaheuristic methods; and (II) to show the promising potential in the combination of both artificial intelligence paradigms. In order to show the capacities of this association, the agent-based model of "Heroes and Cowards" is implemented as a metaheuristic method. Under this scheme, candidate solutions from the metaheuristic approach are considered agents while their interactions are characterized following the behaviors of exploration (distribution) and exploitation (concentration) included in the "Heroes and Cowards" model. To evaluate its performance, the proposed algorithm has been tested in a set of 23 benchmark functions, including multimodal, unimodal, and hybrid benchmark functions. The competitive results indicate that even though agent-based modeling and metaheuristic schemes refer to distinct scientific communities, metaheuristic methods can increase their capabilities through the incorporation of concepts, formalisms and models extracted from agent-based techniques.

This paper is organized as follows: In Section 2, the basic concepts of agent-based modeling are introduced. In Section 3, the model of Heroes and Cowards is reviewed. In Section 4, the proposed metaheuristic method is exposed. In Section 5 the experimental results and the comparative analysis is presented. Finally, in Section 6, conclusions are drawn.

2. Agent-based modeling

Agent-based models [3] corresponds to computational schemes used to explain the behavior of complex systems. Under these models, it is emulated the actions of elements inside the system, also considering the manner these entities influence and are influenced by their environment. Agent-based models are particularly adequate when the behavior of the interacting elements presents an important factor in the results.

Agents correspond to artificial elements programmed to execute prespecified actions [54]. While agents perform their operations based on their behavior, they also cooperate and compete with other elements. The structure of the operations achieved by an agent is straightforward. They cover from easy decisions (such as yes or no actions) to spatial movements.

Classical modeling approaches consider in their scheme only aggregate elements rather than their interactions. Therefore, they are not well suited to represent the behavior of complex systems. Even models with elementary elements and simple interactions can produce behaviors that cannot be generated and analyzed from the perspective of classical models (without interactions) [55]. Instead of this, agent-based models explicitly involve individual interactions representing their effects in the system results.

Most of the models based on agent-based models do not use difficult behavioral models or sophisticated architectures. In spite of their simplicity, these models allow producing different complex behavioral patterns as a result of the interactions generated among the set of simple agents [56]. Global behavioral patterns correspond to identifiable distributions that represent macroscopic spatial regularities. In agent-based models, virtual elements make decisions considering programmed rules [57]. The rules characterize the behavior of each individual in an abstract way. It is relatively simple to describe the interactions among the agents once the relevant elements of the system have been identified. Different types of information can be included in the rules such as qualitative information, quantitative data and expert opinions. In rule construction, the idea is to find a trade-off between accuracy and simplicity. The rules need to be simple that they can capture the main theoretical elements of the system [58]. Although the emphasis on model design is to maintain the rules as simple as possible, it is also determinant to guarantee that the rules meet the required accuracy level. However, too much detail can be counter-productive since it makes difficult the observation of the relationship between the agent

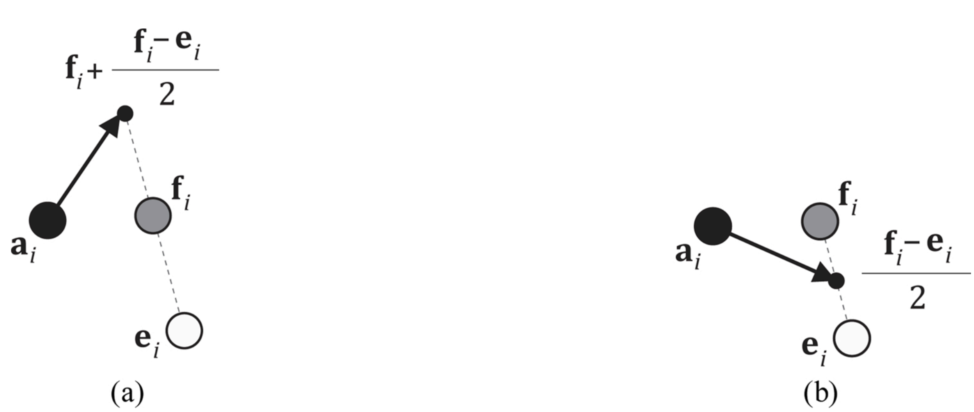

Fig. 1. Effect of the rules that control (a) the coward and (b) heroical behavior.

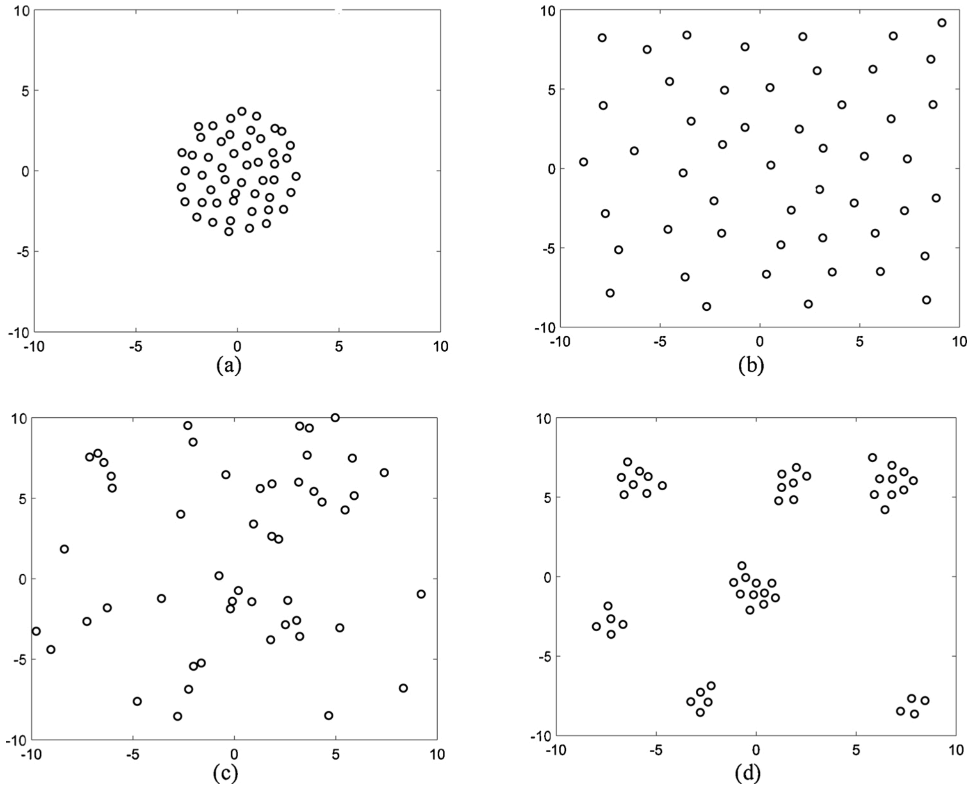

Fig. 2. Examples of global patterns produced by the operation of the Heroes and Cowards model. (a) Initial distribution, (b) global pattern produced by the model in the coward phase considering as initial configuration the provided by (a), (c) initial distribution and (d) final pattern generated by the model in the heroical phase assuming as an initial distribution the agents shown in (c).

and its corresponding behavior.

A generic agent-based approach involves the following stages. Firstly, it is initialized a group of A agents $\{\mathbf{a}_1, \ldots, \mathbf{a}_A\}$. In this step, all agents are set in a specific state or location. Afterward, randomly or considering a particular order, each element $\mathbf{a}_i$ ($i \in 1, \ldots, A$) is selected. Then, this specific agent $\mathbf{a}_i$ is undergone to a set of rules which modify its location, state and relationship with other agents. These rules consist of a set of relations and conditions dictated by local influences (neighbor elements) or other agents. This process is executed until a certain stop criterion has been attained.

3. Heroes and cowards

In the complex system community, there is an agent-based model known as "Heroes and Cowards" [1,12–14] used to illustrate how simple rules can produce complex collective behaviors that are very difficult to reproduce by employing classical modeling techniques. The model produces complex global patterns of concentration and distribution through the interaction of agents that follows simple behavioral rules.

In Heroes and Cowards, a set of A agents $\{\mathbf{a}_1, \ldots, \mathbf{a}_A\}$ are initialized with a random position in a two-dimensional space. Each agent $\mathbf{a}_i$ selects another agent $\mathbf{a}_p$ as its "friend" $\mathbf{f}_i$ and another $\mathbf{a}_q$ as its "enemy" $\mathbf{e}_i$ where $i, p, q \in (1, \ldots, A)$ and $i \neq p \neq q$. The friend $\mathbf{f}_i$ and enemy $\mathbf{e}_i$ selected by $\mathbf{a}_i$ maintain this association during the complete simulation. The model consists of two phases. In the coward stage, every agent behaves as "coward". Under this condition, the agent $\mathbf{a}_i$ moves so that the friend $\mathbf{f}_i$ is always located between $\mathbf{a}_i$ and the enemy $\mathbf{e}_i$ (in a cowardly manner, hiding from its enemy behind its friend). The rule that controls this behavior is formulated as follows:

$$\mathbf{a}_i(t+1) = (1-\beta)\cdot\mathbf{a}_i(t) + \beta\left(\mathbf{f}_i + \left(\frac{\mathbf{f}_i - \mathbf{e}_i}{2}\right)\right) \tag{1}$$

Where t corresponds to the current iteration and β ($\beta \in [0, 1]$) refers to a factor that determines the velocity with which the agent $\mathbf{a}_i$ is displaced. This behavior is illustrated in Fig. 1(a).

In the heroical phase, each agent behaves as "hero". Therefore, the agent presents a strategy where $\mathbf{a}_i$ is moved in a position between its friend $\mathbf{f}_i$ and enemy $\mathbf{e}_i$ (protecting the friend from the enemy in a heroically). The behavioral rule that determines this interaction is formulated as follows:

$$\mathbf{a}_i(t+1) = (1-\beta)\cdot\mathbf{a}_i(t) + \beta\left(\frac{\mathbf{f}_i - \mathbf{e}_i}{2}\right) \tag{2}$$

This behavior is illustrated in Fig. 1(b). The Heroes and Cowards model considers an artificial moderator that decides according to a specific number of iterations N_t when the agents behave as cowards (Eq. (1)) or heroes (Eq. (2)). Therefore, in the model, the phases are performed intercalated. N_t iterations last the coward phase while the heroical phase considers the next N_t iterations. This process continues until a determined number of phases have been reached.

Like any other agent-based model, the agents in Heroes and Cowards updates their position (state) in each iteration. Under such conditions, the relation of each agent $\mathbf{a}_i$ with its related friend $\mathbf{f}_i$ and enemy $\mathbf{e}_i$ is dynamic. Therefore, the model produces complex spatial behaviors of concentration and distribution through the interaction of all agents. During the coward phase, agents are distributed along the space as a consequence of the scape process from the enemy. On the other hand, during the hero stage, agents semi-concentrate around positions marked by the agent distributions.

Fig. 2 presents examples of global patterns produced by the operation of the Heroes and Cowards model. In the Figure, the model is simulated by using a set of 50 agents ($A = 50$). Fig. 2 (b) shows the final obtained pattern by the model in its coward phase after 100 iterations considering as initial configuration the distribution shown in Fig. 2(a). As it can be seen, agents, initially concentrated in the center, are distributed themselves along the space. Fig. 2(d) represents the final obtained behavioral pattern by the heroical phase after 100 iterations considering as initial configuration the distribution shown in Fig. 2(c). A simple inspection from the Figure indicates that agents originally distributed in the space make semi concentrations in regions of the two-dimensional space.

4. An agent-based approach as a metaheuristic method

Although all metaheuristic schemes emulate very different processes or systems, the operators used to model individual behavior are very similar. The idea behind the design of many metaheuristic methods is to configure a recycled set of rules that has demonstrated to be successful in previous approaches for producing new optimization schemes. Such common rules have been designed without considering the final global pattern obtained by the individual interactions.

Different from metaheuristics, agent-based modeling aims to relate the global behavioral patterns produced by the collective interaction of the individuals with the set of rules that describe their behavior. Under this perspective, several agent-based modeling techniques that generate very complex search global behaviors can be used to produce or improve efficient optimization algorithms.

In this paper, we highlight the relationship between metaheuristic schemes and agent-based modeling. In order to show the capacities of this association, the agent-based model of "Heroes and Cowards" is implemented as a metaheuristic method. The section is divided into three parts: (4.1) Problem formulation, (4.2) the description of the agent-based model of heroes and cowards as a metaheuristic method and (4.3) the computational procedure.

4.1. Problem formulation

An optimization method is designed to find a global solution for a nonlinear problem with box constraints according to the following formulation [47]:

$$\text{Maximize}/J(\mathbf{x})\mathbf{x} = (x_1, \ldots, x_d) \in \mathbb{R}^d \tag{3}$$

Minimize

subject to$\mathbf{x} \in \mathbf{X}$

Where $J : \mathbb{R}^d \rightarrow \mathbb{R}$ corresponds to a d -dimensional nonlinear function and $\mathbf{X}$ represents a constrained search space $\left(\mathbf{x} \in \mathbb{R}^d | l_i \leq x_i \leq u_i, i = 1, \ldots, d\right)$ by the lower (l_i) and upper (u_i) bounds.

To solve the optimization problem formulated by Eq. 3, from a metaheuristic perspective, a population of $\mathbf{A}^k$ $\left(\{\mathbf{a}_1^k, \ldots, \mathbf{a}_N^k\}\right)$ of N candidate solutions (agents) evolves from a starting point ($k = 1$) to a *gen* number of iterations ($k = gen$). In the population, each agent $\mathbf{a}_i^k$ ($i \in [1, \ldots, N]$) represents a d -dimensional vector $\{a_{i,1}^k, \ldots, a_{i,d}^k\}$, which corresponds to the decision variables involved by the optimization problem. In the first iteration, the metaheuristic method starts generating a group of N agents with values uniformly distributed within the pre-specified lower (l_i)) and upper (u_i) bounds. Then, at each iteration, a determined number of metaheuristic operations are applied over the agents of the population $\mathbf{A}^k$ to produce the new population $\mathbf{A}^{k+1}$. The quality of each individual $\mathbf{a}_i^k$ is evaluated in terms of its solution regarding the objective function $J(\mathbf{a}_i^k)$ whose result represents the fitness value of $\mathbf{a}_i^k$. As the search strategy evolves, the best current agent $\mathbf{b}$ $\{b_1, \ldots, b_d\}$ is preserved, since $\mathbf{b}$ corresponds to the best available solution seen so-far.

4.2. Heroes and cowards as a metaheuristic method

This subsection explains the way in which the agent-based approach of "Heroes and cowards" has been adapted to perform as a competitive optimization method. The method considers three elements: (4.2.1) Initialization, (4.2.2) operators and (4.2.3) phase management.

4.2.1. Initialization

In the first iteration ($k = 1$), the method starts generating a set of A agents $\mathbf{A}^k = \{\mathbf{a}_1^k, \ldots, \mathbf{a}_N^k\}$ with random positions in a d -dimensional space $\left(\mathbf{a}_i^k = \{a_{i,1}^k, \ldots, a_{i,d}^k\}\right)$. In this process, each decision variable $a_{i,j}^k$ ($i \in 1, \ldots, N; j = 1, \ldots, d$) that corresponds to the j -th parameter of the i -th agent is set with a numerical value uniformly determined between the defined lower (l_i) and upper (u_i) limits, so that

$$a_{i,j}^k = l_i + \text{rand}(0,1) \cdot (u_i - l_i) \tag{4}$$

Each agent $\mathbf{a}_i^k$ selects another agent $\mathbf{a}_p^k$ as its "friend" $\mathbf{f}_i^k$ and another $\mathbf{a}_q^k$ as its "enemy" $\mathbf{e}_i^k$ where $i, p, q \in (1, \ldots, A)$ and $i \neq p \neq q$. The friend $\mathbf{f}_i^k$ and enemy $\mathbf{e}_i^k$ selected by $\mathbf{a}_i^k$ maintain this association during the complete process.

4.2.2. Operators

The "Heroes and cowards" model consists of two processes: Coward and heroical phases. Under such conditions, the proposed metaheuristic scheme implements an operator for each phase. Each operator updates the position of an agent $\mathbf{a}_i^k$ in relation to the position of its friend $\mathbf{f}_i^k = \{f_{i,1}^k, \ldots, f_{i,d}^k\}$ and enemy $\mathbf{e}_i^k = \{e_{i,1}^k, \ldots, e_{i,d}^k\}$. Such operations are practically the same as the behavioral rules involved in the original model, with only an adaptation. This modification represents the incorporation of a random number in order to add a stochastic effect in the search strategy. Therefore, the coward operator is defined as follows:

$$a_{i,j}^{k+1} = (1-\beta) \cdot a_{i,j}^k + \beta\left(f_{i,j}^k + \left(\frac{f_{i,j}^k - e_{i,j}^k}{2}\right)\right) + \alpha \cdot \text{rand}(-1,1) \tag{5}$$

Where α represents the intensity of the stochastic effect and rand$(-1,1)$ a function that delivers a random number uniformly distributed between -1 and 1. On the other hand, the heroical operator is modeled as follows:

$$a_{i,j}^{k+1} = (1-\beta) \cdot a_{i,j}^k + \beta \cdot \left(\frac{f_{i,j}^k - e_{i,j}^k}{2}\right) + \alpha \cdot \text{rand}(-1,1) \tag{6}$$

In the adaptation of the heroes and coward model as an optimization scheme, the location of all agents is updated by the coward or heroical operators except for the best element $\mathbf{b}$ of the population $\mathbf{A}^k$. In case of a maximization problem, this agent will be selected in each iteration k so that

$$\mathbf{b} = \arg\max_{\mathbf{a}_i^k \in \mathbf{A}^k} J(\mathbf{a}_i^k) \tag{7}$$

Once obtained, this agent $\mathbf{b}$ is not modified by the operators.

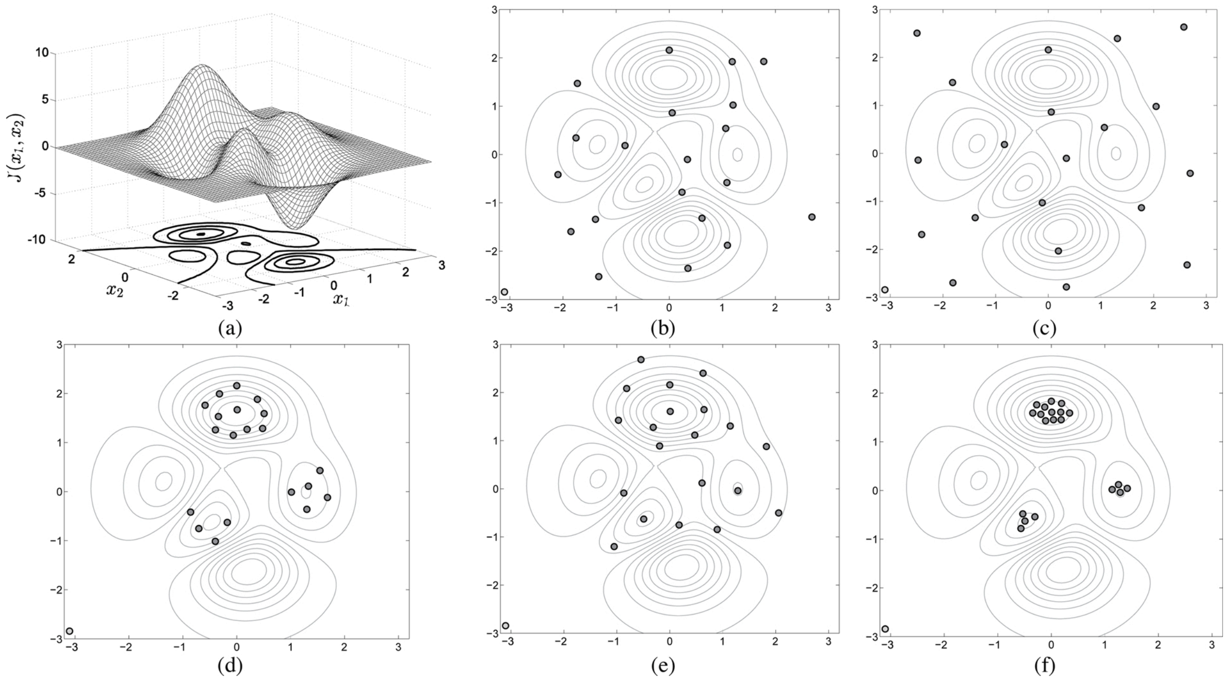

Fig. 3. "Heroes and cowards" model behavior considering 100 iterations when it solves the optimization problem formulated by Eq. (8). (a) objective function with its respective contour, (b) 1 iteration (initialization), (c) 20 iterations (first coward phase), (d) 40 iterations (First heroical phase), (e) 60 iterations (second coward phase) and (f) 80 iterations (second heroical phase).

4.2.3. Phase management

The position of each agent $\mathbf{a}_i^k$ (except $\mathbf{b}$) is modified iteratively according to one of the operators while the phase of the model has not been changed. Similar to the original model, the phases are performed intercalated. N_t iterations last the coward phase (Eq. (5)) while the heroical phase (Eq. (6)) considers the next N_t iterations. This process continues until a stop criterion has been reached.

4.3. Computational procedure

The adapted "Heroes and cowards" model has been implemented as an iterative scheme that considers some processes in its operation. In the form of pseudo-code, Algorithm 1 summarizes the operations of the whole process. The approach requires as input data the number of agents A, the displacement velocity β, the intensity of the stochastic effect α, the number of iterations of each phase N_t and the maximum number of executions *gen* (line 1). As another metaheuristic scheme, initially (line 2), the method produces a set of (A) agents with positions uniformly distributed between the pre-specified limits. Such agents correspond to the initial population $\mathbf{A}^1$. Then, the best element $\mathbf{b}$ from $\mathbf{A}^1$ regarding its fitness value is selected (Line 3). The method begins considering N_t iterations of the coward phase (line 4). Therefore, during this phase each agent $\mathbf{a}_i^k$ except the best element $\mathbf{b}$ is modified (line 7) by the operator defined in Eq. (5). Once the N_t iterations have been reached, (line 14) a change of phase is achieved (line 15). After changed the phase, the heroical phase is conducted. During this phase each agent $\mathbf{a}_i^k$ except the best element $\mathbf{b}$ is modified (line 10) by the operator defined in Eq. (6). After the application of an operator, the best element $\mathbf{b}$ from $\mathbf{A}^k$ regarding its fitness value is selected (Line 12). This process is conducted until the maximal number of generations *gen* has been reached. As output (line 19), the algorithm delivers as output the last obtained, since it represents the final solution.

1. **Input:** $A, \beta, \alpha, N_t, gen$
2. $\mathbf{A}^1 \leftarrow$ **Initialize**(A);
3. $\mathbf{b} \leftarrow$**SelectBestAgent**($\mathbf{A}^1$);
4. Phase= Coward; m=1;
5. **while** k<= gen **do**
6. **If** (Phase== Coward)
7. $\mathbf{A}^{k+1} \leftarrow$ **CowardOperator**($\forall \mathbf{a}_i^k \in \mathbf{A}^k$where $\mathbf{a}_i^k \neq \mathbf{b}$);
8. **end if**
9. **If** (Phase== Heroical)
10. $\mathbf{A}^{k+1} \leftarrow$ **HeroicalOperator**($\forall \mathbf{a}_i^k \in \mathbf{A}^k$where $\mathbf{a}_i^k \neq \mathbf{b}$);
11. **end if**
12. $\mathbf{b} \leftarrow$**SelectBestAgent**($\mathbf{A}^{k+1}$);
13. m=m+1; $k = k + 1$;
14. **If** (m== N_t)
15. Phase← **ChangeOfPhase**;
16. m=1;
17. **end if**
18. end **while**
19. **Output: b**

Algorithm 1. Summarized processes of the adapted "Heroes and cowards" model.

In order to illustrate the operation of the "Heroes and cowards" model, an optimization example is carried out. The example aims to detect the maximal value of the two-dimensional objective function $J(x_1, x_2)$ defined in Eq. (8).

$$J(x_1,x_2) = 3(1-x_1)^2 e^{-(x_1^2-x_2^2)} - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right) e^{\left(-x_1^2-x_2^2\right)} - 1/3e^{\left(-(x_1+1)^2-x_2^2\right)} \tag{8}$$

In the example, the algorithm has been set with the following parameters: $A = 20$, $\beta = 0.7$, $\alpha = 0.3$, $N_t = 20$, $gen = 100$. Assuming these parameters, all the agents $\{\mathbf{a}_i^1 = (a_{i,1}^1, a_{i,2}^1)\}$ from $\mathbf{A}^1$ are initialized with random values uniformly distributed within the interval of $3 \leq x_1, x_2 \leq 3$. In the coward phase of the search strategy, the scheme promotes the distribution and exploration of solutions along the search space. On the other hand, in the heroical phase, the semi concentration of solutions is produced through the use of attraction movements. As the iterations progress, in the heroical phase, the exploitation is intensified to refine the quality of their solutions.

Fig. 3 shows the behavior of the "Heroes and cowards" model. Fig. 3 (a) exhibits the objective function to optimize $J(x_1, x_2)$ with its respective contour representation. During the evolution of the algorithm, five points, (b) 1 iteration (initialization), (c) 20 iterations (first coward phase), (d) 40 iterations (First heroical phase), (e) 60 iterations (second coward phase) and (f) 80 iterations (second heroical phase), have been selected to show its operation. Point (b) represents an early stage of the algorithm where the elements are almost in their initial random location. As can be seen in Fig. 3(c)-(e), the model produces groups as the iterations increase, until all elements converge in Fig. 3(f) to the global and local maxima.

5. Experimental results

Metaheuristic optimization techniques have been proposed as stochastic algorithms to solve optimization problems where classical methodologies are not suitable to operate since real-world optimization problems contain multiple optima. To evaluate the performance of metaheuristic schemes, the scientific community has proposed a set of benchmark functions to numerically expose the performance of such methods. In this study, the performance of metaheuristic methodologies is evaluated over a standard set of 23 mathematical benchmark functions [59,60]. Such a benchmark set contains functions with different complexities to measure the precision, robustness and scalability of those mechanisms. Additionally, three engineering design problems commonly found in the related literature [61,62] are used to evaluate the capabilities of the proposed method by solving real-world optimization scenarios. During the optimization process, the algorithms are evaluated considering the maximum number of generations (*gen*) as a stop criterion. This criterion has been extensively used in the metaheuristic optimization domain.

This section presents the numerical results of the proposed "Heroes and cowards" model, which for identification proposes will be called as EA−HC. This model, as a metaheuristic algorithm, is evaluated using a set of benchmark functions as well as engineering design optimization problems. In Appendix A, Table A1 mathematically describes the set of test functions used in the performance analysis. In the table, n corresponds to the n-dimensional vector at which the test functions are evaluated, $f(x^*)$ represents the optimal value of a given function evaluated at position x^* and $\boldsymbol{S}$ corresponds to the search space, defined by the lower and upper limits of the search space. To prove the scalability of the proposed method, the evaluation for each test function is operated by 30, 100 and 200-dimensional search spaces.

The performance results exposed by the proposed method are compared against the performance results of 9 evolutionary methodologies, named; Artificial Bee Colony (ABC) [18], Differential Evolution (DE) [19], Particle Swarm Optimization (PSO) [36], Cuckoo Search (CS) [43], Differential Search (DS) [20], Adaptive Differential Evolution With Optional External Archive (JADE) [23], Moth-Flame Optimization (MFO) [25], Multi-Verse Optimizer (MVO) [21], and the Sine Cosine Algorithm (SCA) [22]. Also, the numerical comparison among EA−HC method and the rest of competitors considering design optimization problems is analyzed. Each real-world design problem includes the well-known Three-bar truss design, Tension/compression spring design and Welded beam design. In Appendix B, the Tables B1, B2, and B3 mathematically describe each optimization problem, respectively.

The experimental results are divided into four sub-sections. In the first section (5.1), the performance of the proposed algorithm is evaluated with regard to its tuning parameters. In the second section (5.2), the overall performance of the proposed method is compared to different popular metaheuristic algorithms is provided.

Table 1
Parameter configuration for each metaheuristic method used in the experimental study.

Algorithm	Parameter(s)	Reference
ABC	The population size has been set as $limit = 50$	[18]
DE	The variant considered is DE/rand/bin, where crossover probability $CR = 0.5$, mutation rate $F = 0.6$ and differential weight $dw = 0.2$	[19]
PSO	The parameters are set to $c_1 = 2$ and $c_2 = 2$ with linearly decreasing weight factor from 0.9 to 0.2	[36]
CS	The discover probability is set to $pa = 0.25$	[43]
DS	The algorithm has been implemented considering the scale parameter $cp = 0.5$ for the Gamma distribution	[20]
JADE	The variant considered is ldquoDE/current-to- p bestrdquo, where crossover probability $cr = 0.5$ and differential weight $dw = 0.6$	[23]
MFO	The source code has been obtained by its reference	[25]
MVO	The source code has been obtained by its reference	[21]
SCA	The implementation considers the guidelines described by the author	[22]
EA-HC	$\beta = 0.7$, $\alpha = 0.3$ and $N_t = 50$.	

In the third Section (5.3), the convergence analysis for each test function considering each metaheuristic approach is presented. Finally, in the fourth Section (5.4) the ability of the "heroes and coward" model to solve engineering problems is analyzed.

5.1. Performance evaluation with regard to its own tuning parameters

The two parameters β and α present a determinant influence in the expected performance of the EA−HC scheme. In this sub-section is analyzed the behavior of the proposed scheme considering different configurations of these parameters. In the test, one factor-at-a-time of the two parameters is tested while the other element remains fixed to a default value. To minimize the stochastic effect, each benchmark function is executed independently for 30 times. As a termination criterion, the maximum number of iterations is considered. It has been set to 1000. In all simulations, the population size is fixed to 50 individuals.

First, the behavior of the proposed algorithm is analyzed, considering different values for β. In the analysis, the values of β are varied from 0 to 1, whereas the value of α remains fixed to 0.3. In the simulation, the proposed method is executed independently 30 times for each value of β, on each benchmark function. Then, the performance of the proposed algorithm is evaluated, considering different values for α. In the experiment, the values of α are varied from 0.0 to 0.5, whereas the value of β remains fixed to 0.7. The obtained results suggest that a proper combination of the parameter values can improve the performance of the proposed method and the quality of solutions. With the experiment can be concluded that the best parameter set is composed of the following values: $\beta = 0.7$ and $\alpha = 0.3$. They are kept for the next experiments.

5.2. Performance comparison

In this section, the performance of EA−HC is analyzed and numerically compared in terms of the fitness value (in this study, it is considered the performance results for minimization) against nine well-known evolutionary approaches considering a set of 23 test functions. The selected test functions include uni-modal, multi-modal and hybrid benchmark functions. To make a fair comparison among evolutionary methods, the evolutionary process of each algorithm uses $gen = 1000$ as a stop condition. This condition has been chosen to maintain compatibility with the numerical results of most of the related and novel works [56–58] in the literature. To prove the scalability of the proposed methods, the simulations are evaluated in $n = 30$, $n = 100$ and $n = 200$ dimensions and each experiment has been executed by 30 runs. Since metaheuristic methods are stochastic search methods, statistical

Table 2
Minimization results of benchmark functions of Table A1 with $n = 30$.

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
f_1	f_{Best}	1.05E-12	7.28E+03	8.23E+02	4.78E-03	1.88E-06	2.68E-20	1.94E-06	1.13E-01	9.45E-07	1.47E-144
	f_{Worst}	7.08E-11	1.74E+04	3.32E+03	1.67E-02	2.40E-04	5.45E-06	1.00E+04	2.92E-01	9.22E-02	5.47E-05
	$\bar{f}$	1.52E-11	1.23E+04	1.66E+03	9.27E-03	1.99E-05	4.84E-07	1.20E+03	1.84E-01	6.52E-03	1.89E-06
	σ_f	1.72E-11	2.38E+03	6.65E+02	2.85E-03	4.32E-05	1.22E-06	3.28E+03	4.55E-02	1.83E-02	9.98E-06
f_2	f_{Best}	4.14E-07	3.99E+01	2.31E+01	6.05E-01	7.66E-05	2.69E-05	5.58E-05	1.98E-01	2.43E-09	**3.63E-77**
	f_{Worst}	2.22E-06	6.60E+01	1.03E+02	5.54E+00	1.27E-03	1.66E+00	8.00E+01	5.45E-01	6.03E-05	4.39E-02
	$\bar{f}$	1.13E-06	5.20E+01	4.16E+01	1.67E+00	3.45E-04	2.37E-01	2.85E+01	3.16E-01	6.33E-06	1.74E-03
	σ_f	4.77E-07	6.63E+00	1.89E+01	1.22E+00	2.78E-04	5.16E-01	1.97E+01	8.69E-02	1.29E-05	8.01E-03
f_3	f_{Best}	6.00E+03	4.51E+04	2.13E+03	2.59E+02	9.98E+02	1.21E+04	2.31E+02	9.17E+00	4.36E+01	**5.88E-40**
	f_{Worst}	1.56E+04	7.93E+04	2.03E+04	6.77E+02	4.97E+03	3.53E+04	4.00E+04	3.64E+01	1.10E+04	1.44E+03
	$\bar{f}$	1.14E+04	5.79E+04	7.96E+03	4.37E+02	2.91E+03	2.31E+04	1.56E+04	1.86E+01	3.48E+03	8.31E+01
	σ_f	2.75E+03	9.38E+03	3.64E+03	9.09E+01	1.28E+03	5.87E+03	1.05E+04	7.64E+00	3.42E+03	3.11E+02
f_4	f_{Best}	1.83E+01	5.95E+01	1.50E+01	2.21E+00	2.76E+00	2.67E-03	3.25E+01	3.11E-01	1.20E+00	**4.58E-140**
	f_{Worst}	5.60E+01	7.65E+01	3.50E+01	4.79E+00	9.96E+00	5.15E+01	8.01E+01	1.14E+00	2.50E+01	9.81E-28
	$\bar{f}$	4.46E+01	6.86E+01	2.23E+01	3.39E+00	5.67E+00	6.51E+00	5.65E+01	6.28E-01	1.15E+01	3.27E-29
	σ_f	8.00E+00	3.73E+00	4.30E+00	6.72E-01	1.90E+00	1.57E+01	9.87E+00	2.07E-01	6.61E+00	1.79E-28
f_5	f_{Best}	1.91E+01	2.75E+01	1.40E+02	2.31E+01	2.24E+01	1.29E+01	9.12E+00	2.34E+01	2.70E+01	**5.25E-03**
	f_{Worst}	2.72E+01	2.89E+01	1.05E+03	2.55E+01	7.89E+01	2.63E+01	9.01E+04	8.14E+01	2.87E+01	2.87E+01
	$\bar{f}$	2.44E+01	2.88E+01	3.68E+02	2.47E+01	2.66E+01	1.97E+01	1.57E+04	3.03E+01	2.79E+01	2.17E+01
	σ_f	1.78E+00	4.66E-02	1.87E+02	5.80E-01	9.92E+00	5.20E+00	3.39E+04	1.34E+01	4.46E-01	1.19E+01
f_6	f_{Best}	**0.00E+00**	8.45E+03	7.65E+02	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.00E+00	**0.00E+00**	**0.00E+00**
	f_{Worst}	0.00E+00	1.70E+04	3.34E+03	0.00E+00	0.00E+00	0.00E+00	1.00E+04	1.70E+01	2.00E+00	1.00E+01
	$\bar{f}$	0.00E+00	1.19E+04	1.93E+03	0.00E+00	0.00E+00	0.00E+00	1.60E+03	6.07E+00	6.67E-02	3.33E-01
	σ_f	0.00E+00	2.24E+03	6.78E+02	0.00E+00	0.00E+00	0.00E+00	3.70E+03	3.12E+00	3.65E-01	1.83E+00
f_7	f_{Best}	9.31E-02	3.56E+00	2.16E+00	1.59E-02	1.41E-02	5.20E-02	3.01E-02	6.10E-03	3.22E-03	**5.05E-06**
	f_{Worst}	3.07E-01	1.57E+01	1.14E+01	6.45E-02	6.47E-02	2.14E-01	1.89E+01	3.71E-02	1.12E-01	2.14E-02
	$\bar{f}$	2.21E-01	9.27E+00	5.38E+00	3.36E-02	3.77E-02	1.09E-01	1.84E+00	1.38E-02	2.48E-02	1.04E-03
	σ_f	5.28E-02	2.57E+00	2.34E+00	1.10E-02	1.42E-02	3.90E-02	3.91E+00	6.42E-03	2.21E-02	3.89E-03
f_8	f_{Best}	**2.07E+00**	5.18E+00	4.63E+00	**2.07E+00**	**2.07E+00**	**2.07E+00**	2.08E+00	**2.07E+00**	2.23E+00	**2.07E+00**
	f_{Worst}	2.07E+00	9.68E+00	1.25E+01	2.07E+00	2.08E+00	2.07E+00	2.13E+00	2.07E+00	2.67E+00	3.25E+00
	$\bar{f}$	2.07E+00	7.40E+00	8.09E+00	2.07E+00	2.07E+00	2.07E+00	2.10E+00	2.07E+00	2.36E+00	2.12E+00
	σ_f	2.65E-08	1.01E+00	1.93E+00	4.47E-06	2.60E-03	8.50E-04	1.26E-02	6.13E-05	9.78E-02	2.19E-01
f_9	f_{Best}	**7.13E-02**	3.42E+00	6.05E-01	**7.13E-02**	**7.13E-02**	**7.13E-02**	8.32E-02	7.15E-02	2.16E-01	**7.13E-02**
	f_{Worst}	7.13E-02	6.50E+00	2.60E+00	7.13E-02	7.40E-02	7.13E-02	1.33E-01	7.16E-02	6.32E-01	6.51E+00
	$\bar{f}$	7.13E-02	4.49E+00	1.52E+00	7.13E-02	7.20E-02	7.13E-02	9.99E-02	7.15E-02	3.57E-01	3.83E-01
	σ_f	1.06E-10	7.30E-01	6.29E-01	6.66E-07	1.13E-03	3.86E-08	1.27E-02	4.37E-05	1.12E-01	1.26E+00
f_{10}	f_{Best}	8.20E-09	2.45E+02	9.56E+01	6.49E+01	2.56E+00	1.27E+01	6.87E+01	5.78E+01	5.95E-06	**0.00E+00**
	f_{Worst}	1.02E+00	3.06E+02	2.42E+02	1.08E+02	1.83E+01	2.57E+01	2.42E+02	2.09E+02	8.81E+01	2.53E+02
	$\bar{f}$	1.67E-01	2.75E+02	1.74E+02	8.52E+01	8.37E+00	1.79E+01	1.43E+02	1.10E+02	2.02E+01	5.41E+01
	σ_f	3.79E-01	1.47E+01	3.12E+01	1.02E+01	2.59E+00	3.16E+00	4.01E+01	3.58E+01	2.75E+01	9.09E+01
f_{11}	f_{Best}	2.04E-06	1.59E+01	7.60E+00	2.20E+00	2.14E-04	4.28E+00	5.12E-04	1.03E-01	4.16E-05	**8.88E-16**
	f_{Worst}	3.54E-05	1.77E+01	1.47E+01	6.92E+00	4.89E-03	1.98E+01	2.00E+01	2.61E+00	2.02E+01	4.67E-01
	$\bar{f}$	1.55E-05	1.67E+01	1.13E+01	3.91E+00	1.48E-03	1.04E+01	1.39E+01	9.64E-01	1.64E+01	1.56E-02
	σ_f	8.37E-06	4.70E-01	1.66E+00	1.30E+00	1.31E-03	5.79E+00	8.28E+00	7.69E-01	7.50E+00	8.53E-02
f_{12}	f_{Best}	5.15E-11	7.27E+01	8.75E+00	4.79E-02	6.01E-07	9.61E-12	5.09E-06	2.38E-01	2.26E-06	**0.00E+00**
	f_{Worst}	2.80E-05	1.79E+02	2.77E+01	1.81E-01	3.44E-02	1.24E-02	9.09E+01	6.10E-01	6.93E-01	3.15E-03
	$\bar{f}$	1.20E-06	1.12E+02	1.73E+01	1.02E-01	7.08E-03	1.26E-03	1.45E+01	4.47E-01	1.75E-01	1.05E-04
	σ_f	5.13E-06	2.24E+01	4.68E+00	2.77E-02	1.00E-02	3.14E-03	3.35E+01	1.10E-01	2.00E-01	5.75E-04
f_{13}	f_{Best}	**−3.00E+00**	5.39E-02	−3.24E-02	−2.95E+00	**−3.00E+00**	**−3.00E+00**	**−3.00E+00**	−2.85E+00	**−3.00E+00**	**−3.00E+00**
	f_{Worst}	−3.00E+00	1.44E+00	3.92E+00	−2.42E+00	−3.00E+00	−3.00E+00	−6.00E-01	−1.08E+00	−3.00E+00	−3.00E+00
	$\bar{f}$	−3.00E+00	7.96E-01	1.71E+00	−2.72E+00	−3.00E+00	−3.00E+00	−2.59E+00	−2.16E+00	−3.00E+00	−3.00E+00
	σ_f	6.40E-14	3.31E-01	1.01E+00	1.65E-01	5.82E-08	1.95E-05	4.96E-01	3.98E-01	2.65E-05	1.62E-08
f_{14}	f_{Best}	2.75E+00	3.59E+01	3.34E+00	2.75E+00	2.75E+00	2.75E+00	1.97E+01	2.75E+00	2.75E+00	**2.72E+00**
	f_{Worst}	1.34E-07	1.27E+01	8.86E-01	5.95E-06	9.84E-09	2.75E+00	2.26E+01	1.05E-06	2.55E-04	2.75E+00
	$\bar{f}$	2.75E+00	1.49E+01	2.80E+00	2.75E+00	2.75E+00	2.75E+00	2.75E+00	2.75E+00	2.75E+00	2.72E+00
	σ_f	2.75E+00	6.25E+01	7.68E+00	2.75E+00	2.75E+00	6.79E-04	9.97E+01	2.75E+00	2.75E+00	5.59E-03
f_{15}	f_{Best}	9.96E-16	3.10E+00	1.13E+00	1.81E-06	2.97E-10	1.44E-17	7.32E-10	4.76E-05	2.93E-10	**−1.04E+00**
	f_{Worst}	1.39E-14	7.40E+00	3.93E+00	1.03E-05	6.53E-08	6.02E-09	4.00E+00	1.69E-04	3.03E-06	1.33E-14
	$\bar{f}$	4.59E-15	5.05E+00	2.21E+00	3.85E-06	6.86E-09	3.48E-10	1.04E+00	7.72E-05	3.35E-07	−3.45E-02

(*continued on next page*)

Table 2 (*continued*)

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
	σ_f	3.41E-15	9.97E-01	7.48E-01	1.69E-06	1.22E-08	1.15E-09	1.77E+00	2.54E-05	7.04E-07	1.89E-01
f_{16}	f_{Best}	**−1.25E+04**	−5.45E+03	−8.26E+03	−9.31E+03	**−1.25E+04**	−1.06E+04	−1.05E+04	−1.00E+04	−4.83E+03	**−1.25E+04**
	f_{Worst}	−1.20E+04	−3.73E+03	−4.14E+03	−8.38E+03	−1.19E+04	−8.09E+03	−7.20E+03	−5.56E+03	−3.52E+03	−3.83E+03
	$\bar{f}$	−1.22E+04	−4.40E+03	−5.81E+03	−8.65E+03	−1.23E+04	−9.44E+03	−8.68E+03	−7.92E+03	−4.07E+03	−4.95E+03
	σ_f	1.19E+02	3.64E+02	9.44E+02	2.11E+02	1.91E+02	6.93E+02	7.68E+02	8.60E+02	2.85E+02	2.12E+03
f_{17}	f_{Best}	8.06E-14	1.36E+06	1.31E+01	5.40E-01	1.09E-08	3.53E-12	4.87E-07	7.55E-04	3.03E-01	**1.57E-32**
	f_{Worst}	2.12E-12	4.05E+07	3.42E+03	2.18E+00	1.04E-01	8.76E-04	1.98E+00	4.59E+00	1.11E+01	1.12E-09
	$\bar{f}$	5.04E-13	1.64E+07	3.69E+02	1.15E+00	3.46E-03	1.67E-04	2.93E-01	9.20E-01	1.20E+00	4.17E-11
	σ_f	3.97E-13	1.08E+07	9.05E+02	4.23E-01	1.89E-02	2.26E-04	5.15E-01	1.00E+00	1.94E+00	2.05E-10
f_{18}	f_{Best}	5.99E-13	7.68E+06	6.98E+01	5.53E-02	2.36E-08	8.89E-12	2.64E-05	7.68E-03	2.10E+00	**1.35E-32**
	f_{Worst}	1.33E-09	1.02E+08	2.83E+05	2.95E-01	9.49E-05	2.27E-04	3.60E+00	1.21E-01	5.04E+00	1.56E-11
	$\bar{f}$	7.02E-11	4.89E+07	6.62E+04	1.59E-01	6.59E-06	1.78E-05	2.03E-01	3.79E-02	2.73E+00	5.20E-13
	σ_f	2.41E-10	1.87E+07	7.98E+04	6.15E-02	1.72E-05	5.30E-05	6.10E-01	2.22E-02	5.80E-01	2.84E-12
f_{19}	f_{Best}	2.67E-07	5.22E+03	1.21E+03	6.68E-01	2.12E-04	1.63E-06	4.48E-04	3.09E-01	1.09E-06	**6.49E-81**
	f_{Worst}	1.70E-06	1.89E+04	2.23E+04	1.51E+01	2.98E-03	3.55E+00	1.00E+05	7.33E-01	8.34E-03	4.34E+00
	$\bar{f}$	9.72E-07	1.35E+04	4.79E+03	3.64E+00	9.59E-04	1.22E-01	2.53E+04	5.13E-01	1.77E-03	1.45E-01
	σ_f	4.09E-07	3.10E+03	4.07E+03	3.60E+00	7.20E-04	6.48E-01	2.25E+04	1.22E-01	2.11E-03	7.93E-01
f_{20}	f_{Best}	**2.90E+01**	5.15E+02	2.33E+02	8.77E+01	**2.90E+01**	2.91E+01	5.30E+01	9.13E+01	**2.90E+01**	**2.90E+01**
	f_{Worst}	2.90E+01	7.81E+02	4.66E+02	1.23E+02	4.73E+01	4.08E+01	4.01E+02	1.98E+02	2.90E+01	3.04E+01
	$\bar{f}$	2.90E+01	6.30E+02	3.54E+02	1.09E+02	3.09E+01	3.32E+01	1.29E+02	1.45E+02	2.90E+01	2.90E+01
	σ_f	1.00E-05	5.96E+01	5.38E+01	8.76E+00	4.78E+00	3.04E+00	8.78E+01	2.45E+01	4.52E-03	2.59E-01
f_{21}	f_{Best}	2.78E+02	4.96E+07	1.12E+05	6.88E+01	1.56E+02	**3.20E+01**	8.20E+06	5.13E+01	3.13E+02	**3.20E+01**
	f_{Worst}	3.26E+01	1.72E+07	1.64E+05	8.21E+00	4.96E+01	4.78E+02	5.80E+07	6.52E+00	6.28E+02	3.07E+02
	$\bar{f}$	1.92E+02	2.10E+07	1.84E+03	5.93E+01	9.01E+01	1.81E+02	4.79E+01	4.06E+01	3.20E+01	4.20E+01
	σ_f	3.38E+02	7.77E+07	6.82E+05	9.59E+01	2.63E+02	1.16E+02	4.10E+08	6.80E+01	3.55E+03	5.02E+01
f_{22}	f_{Best}	**2.90E+01**	5.45E+02	3.00E+02	1.27E+02	**2.90E+01**	2.92E+01	4.28E+01	8.95E+01	**2.90E+01**	**2.90E+01**
	f_{Worst}	2.90E+01	9.86E+02	2.97E+03	1.84E+02	4.35E+01	4.22E+01	2.53E+03	2.51E+02	2.90E+01	2.90E+01
	$\bar{f}$	2.90E+01	7.46E+02	7.10E+02	1.59E+02	2.99E+01	3.38E+01	9.66E+02	1.66E+02	2.90E+01	2.90E+01
	σ_f	4.15E-06	1.07E+02	5.79E+02	1.50E+01	3.56E+00	2.79E+00	5.61E+02	3.47E+01	3.79E-03	1.91E-04
f_{23}	f_{Best}	−8.38E+01	1.87E+06	1.40E+03	−7.29E+01	−8.31E+01	−8.34E+01	−8.27E+01	−8.17E+01	−3.23E+01	**−8.39E+01**
	f_{Worst}	−8.34E+01	2.82E+07	1.56E+06	−4.76E+01	−8.19E+01	−8.18E+01	1.02E+09	−1.30E+01	5.25E+02	−8.10E+01
	$\bar{f}$	−8.36E+01	9.86E+06	5.81E+04	−6.34E+01	−8.26E+01	−8.25E+01	1.84E+08	−7.68E+01	6.67E+00	−8.38E+01
	σ_f	1.17E-01	6.42E+06	2.83E+05	7.02E+00	3.69E-01	3.55E-01	2.74E+08	1.65E+01	9.99E+01	5.39E-01

validation for the results must also be included in order to eliminate the random effect. In this study, the numerical results have been validated, considering the Wilcoxon rank-sum [63].

The performance of most of the metaheuristic approaches is given by the correct setting step of configuration parameters to improve their search capabilities. Such configurations inherently depend on the optimization problem that wanted to be solved. In the optimization theory, The No-Free-Lunch (NFL) theorem states that there is no single algorithm that can solve any optimization problem. That is, if an algorithm X outperforms algorithm Y for the W optimization problem, maybe algorithm Y outperforms algorithm X for the G optimization problem. Under such circumstances, the design of metaheuristic algorithms can include tuning parameters to increase the possibility of locating optima values efficiently.

In comparison, the parameter configuration for each algorithm has been set according to the values presented in Table 1. These configurations have been selected following the reported guidelines by the own authors. These values correspond to the configuration in which (according to the own authors) the compared method reach their best performance. All the algorithms used in the comparisons have been collected from the authors through public repositories.

For the experimental study, the population size has been set to 50 individuals for each metaheuristic approach. Tables 2–4 contain the performance results of the numerical comparison. Table 2 reports the numerical results considering $n = 30$ dimensions. Table 3 reports the numerical results for $n = 100$ dimensions. Additionally, Table 4 reports the numerical results considering $n = 200$ dimensions. In the tables, the best fitness value is represented as f_{Best}, the average fitness as $\bar{f}$, the standard deviation as σ_f and the worst value as f_{Worst}. The tables also present the best performance entries in boldface.

According to Table 2, it is quite evident that the proposed EA−HC outperforms the rest of the metaheuristic algorithms considered in the comparison study. In the case of functions f_6, f_8, f_9, f_{13},f_{16},f_{20},f_{21}, and f_{22}, the proposed EA−HC performs quite similarly to other evolutionary techniques. For function f_6, ABC, CS, DS, JADE, MFO and SCA achieve similar performance than EA−HC. For function f_8, ABC, CS, DS, JADE, MVO and EA−HC obtain the same best fitness value; however, it can be shown the median value of the fitness value for EA−HC is quite greater than the rest of the median values in this function. Also, this phenomenon is replied in function f_9 where the performance of ABC, CS, DS and JADE presents the same fitness value. Additionally, ABC, DS, JADE, MFO and SCA achieve the same best fitness value then EA−HC in function f_{13}. One of the most distinctive characteristics of EA−HC, is the process of changing between exploration and exploitation stages during the entire optimization process. As a consequence, the mean value of fitness is affected due to this changing mechanism. For the remaining functions, ABC and DS compete directly with the results of EA−HC in function f_{16}. Also, for functions f_{20} and f_{22} the ABC, DS, SCA and EA−HC perform quite similar. Finally, function f_{22} only the JADE method obtains the same results as the proposed algorithm.

By the numerical results of Table 2, it can be demonstrated that EA−HC obtains a better response against the compared metaheuristic approaches for the majority of benchmark functions. Only a few reported results suggest that the performance of the proposed method

Table 3
Minimization results of benchmark functions of Table A1 with $n = 100$.

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
f_1	f_{Best}	3.95E-03	8.61E+04	9.25E+03	1.34E+02	8.15E+01	1.78E-06	2.27E+03	1.69E+01	2.23E+02	**1.05E-105**
	f_{Worst}	1.11E-01	1.18E+05	2.19E+04	3.10E+02	5.41E+02	2.80E-03	5.01E+04	3.45E+01	1.25E+04	1.45E+01
	$\bar{f}$	2.25E-02	1.03E+05	1.55E+04	2.15E+02	1.95E+02	1.58E-04	2.37E+04	2.32E+01	3.39E+03	4.83E-01
	σ_f	2.20E-02	6.98E+03	3.05E+03	4.36E+01	1.16E+02	5.36E-04	1.28E+04	4.28E+00	3.17E+03	2.65E+00
f_2	f_{Best}	6.04E-02	1.35E+07	9.20E+01	1.00E+10	3.26E+00	1.01E-03	8.46E+01	3.19E+02	9.23E-03	**2.15E-91**
	f_{Worst}	1.61E-01	2.82E+19	1.52E+02	1.00E+10	1.18E+01	4.55E+01	3.33E+02	2.36E+16	6.47E+00	3.06E+00
	$\bar{f}$	9.62E-02	1.08E+18	1.17E+02	1.00E+10	6.60E+00	3.43E+00	1.59E+02	1.43E+15	8.98E-01	1.74E-01
	σ_f	2.53E-02	5.15E+18	1.58E+01	0.00E+00	2.04E+00	8.40E+00	4.98E+01	5.47E+15	1.36E+00	6.68E-01
f_3	f_{Best}	1.49E+05	3.82E+05	4.30E+04	3.32E+04	4.66E+04	1.41E+04	1.02E+05	2.30E+04	9.75E+04	**6.89E-149**
	f_{Worst}	2.31E+05	7.80E+05	1.45E+05	6.17E+04	1.13E+05	4.09E+05	3.09E+05	4.55E+04	2.49E+05	5.45E+04
	$\bar{f}$	1.85E+05	5.52E+05	7.65E+04	4.81E+04	7.19E+04	2.72E+05	1.79E+05	3.15E+04	1.69E+05	7.16E+03
	σ_f	1.93E+04	9.79E+04	2.72E+04	7.64E+03	1.86E+04	9.31E+04	5.21E+04	5.18E+03	4.46E+04	1.40E+04
f_4	f_{Best}	8.68E+01	8.16E+01	2.81E+01	1.38E+01	2.85E+01	1.30E+01	8.80E+01	3.11E+01	7.66E+01	**3.01E-133**
	f_{Worst}	9.26E+01	8.90E+01	4.38E+01	2.16E+01	5.53E+01	9.32E+01	9.57E+01	5.64E+01	9.11E+01	2.33E-51
	$\bar{f}$	9.01E+01	8.53E+01	3.61E+01	1.72E+01	4.34E+01	8.58E+01	9.22E+01	4.23E+01	8.50E+01	8.77E-53
	σ_f	1.72E+00	1.52E+00	3.95E+00	2.01E+00	6.01E+00	1.40E+01	1.96E+00	6.57E+00	3.29E+00	4.27E-52
f_5	f_{Best}	9.66E+01	1.26E+03	3.29E+03	9.96E+01	9.73E+01	8.93E+01	2.86E+02	9.16E+01	1.21E+02	**1.75E-01**
	f_{Worst}	1.18E+02	1.70E+03	6.04E+03	1.03E+02	1.64E+02	9.87E+01	6.98E+02	1.86E+02	4.16E+02	9.80E+01
	$\bar{f}$	1.04E+02	1.44E+03	4.62E+03	1.01E+02	1.16E+02	9.24E+01	4.86E+02	1.05E+02	2.34E+02	7.43E+01
	σ_f	6.05E+00	1.11E+02	6.65E+02	1.06E+00	1.97E+01	2.94E+00	1.13E+02	2.24E+01	8.39E+01	4.02E+01
f_6	f_{Best}	4.00E+00	8.90E+04	1.15E+04	3.37E+02	1.24E+02	1.00E+00	3.96E+03	8.50E+01	4.81E+02	**0.00E+00**
	f_{Worst}	2.80E+01	1.19E+05	2.14E+04	5.87E+02	9.14E+02	1.10E+02	4.51E+04	2.34E+02	1.18E+04	0.00E+00
	$\bar{f}$	1.28E+01	1.03E+05	1.64E+04	4.48E+02	3.20E+02	1.97E+01	2.77E+04	1.53E+02	4.20E+03	0.00E+00
	σ_f	6.09E+00	7.81E+03	2.41E+03	7.26E+01	1.87E+02	2.21E+01	1.06E+04	4.12E+01	3.19E+03	0.00E+00
f_7	f_{Best}	1.38E+00	2.32E+02	3.19E+02	3.25E-01	8.16E-01	5.06E-01	4.70E+00	1.36E-01	4.70E+00	**6.98E-06**
	f_{Worst}	2.79E+00	4.55E+02	7.56E+02	8.35E-01	2.35E+00	1.39E+00	4.16E+02	3.88E-01	1.13E+02	2.52E-03
	$\bar{f}$	2.26E+00	3.58E+02	5.01E+02	5.14E-01	1.30E+00	8.80E-01	1.41E+02	2.43E-01	4.86E+01	3.19E-04
	σ_f	3.25E-01	5.24E+01	1.01E+02	1.22E-01	4.04E-01	2.06E-01	1.04E+02	6.36E-02	3.03E+01	5.18E-04
f_8	f_{Best}	2.03E+00	5.72E+01	3.12E+01	5.19E+00	2.03E+00	**2.02E+00**	2.04E+00	2.03E+00	2.11E+00	**2.02E+00**
	f_{Worst}	2.10E+00	6.69E+01	4.47E+01	8.87E+00	2.04E+00	2.03E+00	6.38E+00	2.88E+00	2.29E+00	3.69E+01
	$\bar{f}$	2.05E+00	6.22E+01	3.77E+01	7.03E+00	2.03E+00	2.02E+00	3.18E+00	2.23E+00	2.18E+00	3.53E+00
	σ_f	1.47E-02	2.40E+00	3.37E+00	8.48E-01	3.31E-03	2.78E-03	1.35E+00	2.10E-01	3.57E-02	6.42E+00
f_9	f_{Best}	2.34E-02	4.85E+01	4.05E+01	1.59E-01	3.67E-02	**2.04E-02**	4.54E-02	2.40E-02	9.74E-02	**2.04E-02**
	f_{Worst}	3.65E-02	6.56E+01	5.90E+01	4.77E-01	1.15E-01	2.05E-02	8.11E+00	2.64E-02	4.80E-01	7.35E+00
	$\bar{f}$	2.68E-02	5.66E+01	5.00E+01	3.20E-01	7.09E-02	2.04E-02	1.70E+00	2.53E-02	1.87E-01	4.99E-01
	σ_f	3.05E-03	3.96E+00	4.03E+00	9.18E-02	2.33E-02	1.00E-05	2.75E+00	4.96E-04	7.26E-02	1.82E+00
f_{10}	f_{Best}	4.38E+01	1.14E+03	6.49E+02	4.08E+02	1.62E+02	2.40E+02	5.76E+02	4.43E+02	1.05E+01	**0.00E+00**
	f_{Worst}	8.54E+01	1.27E+03	9.56E+02	5.41E+02	3.54E+02	3.23E+02	8.57E+02	7.29E+02	5.13E+02	9.43E+02
	$\bar{f}$	6.60E+01	1.21E+03	8.19E+02	4.73E+02	2.31E+02	2.77E+02	7.12E+02	5.91E+02	2.08E+02	4.64E+01
	σ_f	1.06E+01	2.88E+01	7.23E+01	3.38E+01	4.46E+01	1.90E+01	6.04E+01	7.69E+01	1.05E+02	1.77E+02
f_{11}	f_{Best}	2.25E+00	1.88E+01	1.21E+01	6.63E+00	3.28E+00	1.69E+01	1.90E+01	2.91E+00	6.52E+00	**8.88E-16**
	f_{Worst}	3.73E+00	1.96E+01	1.46E+01	1.70E+01	1.99E+01	1.99E+01	2.00E+01	1.99E+01	2.06E+01	4.85E-07
	$\bar{f}$	3.03E+00	1.93E+01	1.34E+01	1.09E+01	5.22E+00	1.92E+01	1.98E+01	5.23E+00	1.85E+01	1.99E-08
	σ_f	4.06E-01	1.81E-01	6.22E-01	2.67E+00	3.13E+00	9.37E-01	2.71E-01	4.98E+00	4.63E+00	8.93E-08
f_{12}	f_{Best}	1.74E-02	7.90E+02	8.74E+01	2.26E+00	1.64E+00	8.48E-07	4.26E+01	1.14E+00	6.96E+00	**0.00E+00**
	f_{Worst}	5.92E-01	1.03E+03	1.89E+02	3.95E+00	9.16E+00	1.53E-01	5.65E+02	1.24E+00	1.31E+02	1.02E-09
	$\bar{f}$	1.96E-01	9.16E+02	1.41E+02	2.94E+00	3.05E+00	1.01E-02	2.32E+02	1.20E+00	4.14E+01	3.39E-11
	σ_f	1.55E-01	6.06E+01	2.70E+01	4.24E-01	1.77E+00	2.95E-02	1.27E+02	2.17E-02	3.45E+01	1.85E-10
f_{13}	f_{Best}	**−1.00E+01**	6.69E+00	1.64E+01	−8.65E+00	−9.54E+00	−8.96E+00	−6.52E+00	−6.72E+00	−9.97E+00	**−1.00E+01**
	f_{Worst}	−9.70E+00	1.13E+01	2.66E+01	−6.25E+00	−7.32E+00	−7.04E+00	−6.82E-01	−3.32E+00	−7.39E+00	−1.00E+01
	$\bar{f}$	−9.90E+00	9.86E+00	2.09E+01	−7.23E+00	−8.63E+00	−8.24E+00	−3.64E+00	−5.23E+00	−9.09E+00	−1.00E+01
	σ_f	9.79E-02	9.98E-01	2.64E+00	5.83E-01	5.02E-01	4.93E-01	1.54E+00	8.49E-01	8.30E-01	8.66E-10
f_{14}	f_{Best}	2.73E+00	6.02E+02	1.50E+01	3.22E+00	2.76E+00	2.73E+00	6.05E+01	2.73E+00	5.19E+00	**2.72E+00**
	f_{Worst}	5.48E+00	1.23E+03	5.94E+01	4.13E+00	5.49E+00	6.00E+00	6.84E+02	2.73E+00	2.42E+02	2.73E+00
	$\bar{f}$	3.40E+00	8.62E+02	2.50E+01	3.69E+00	3.18E+00	2.84E+00	2.33E+02	2.73E+00	5.77E+01	2.72E+00
	σ_f	6.22E-01	1.42E+02	9.54E+00	2.70E-01	5.71E-01	5.97E-01	1.46E+02	1.41E-05	6.60E+01	3.34E-03
f_{15}	f_{Best}	1.67E-04	3.23E+01	1.76E+01	5.88E-02	3.03E-02	3.97E-10	8.47E-01	5.81E-03	1.11E-01	**1.06E-165**
	f_{Worst}	4.82E-03	4.67E+01	3.00E+01	1.34E-01	2.44E-01	1.95E-04	2.06E+01	1.13E-02	5.85E+00	1.13E-01
	$\bar{f}$	9.76E-04	3.94E+01	2.37E+01	8.51E-02	8.08E-02	1.38E-05	1.08E+01	8.66E-03	1.47E+00	3.76E-03

(*continued on next page*)

Table 3 (*continued*)

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
	σ_f	9.26E-04	3.14E+00	3.72E+00	1.96E-02	4.47E-02	4.45E-05	5.57E+00	1.29E-03	1.42E+00	2.06E-02
f_{16}	f_{Best}	−3.64E+04	−8.99E+03	−9.27E+03	−2.33E+04	−3.14E+04	−2.57E+04	−2.90E+04	−2.74E+04	−8.19E+03	**−4.19E+04**
	f_{Worst}	−3.43E+04	−6.61E+03	−5.72E+03	−2.11E+04	−2.74E+04	−1.93E+04	−2.03E+04	−2.20E+04	−6.58E+03	−7.35E+03
	$\bar{f}$	−3.52E+04	−7.86E+03	−7.61E+03	−2.19E+04	−2.91E+04	−2.19E+04	−2.38E+04	−2.44E+04	−7.34E+03	−1.81E+04
	σ_f	6.17E+02	6.87E+02	1.05E+03	4.63E+02	1.06E+03	2.19E+03	2.62E+03	1.56E+03	3.99E+02	1.47E+04
f_{17}	f_{Best}	3.28E-05	2.47E+08	2.58E+04	4.40E+00	2.74E+00	2.22E+00	3.93E+04	2.58E+04	2.51E+07	**4.71E-33**
	f_{Worst}	3.11E-02	6.70E+08	9.29E+05	8.46E+00	1.42E+01	7.81E+00	5.22E+08	9.29E+05	3.11E+08	4.20E-09
	$\bar{f}$	1.26E-03	4.48E+08	2.84E+05	5.67E+00	6.58E+00	4.91E+00	1.08E+08	2.84E+05	1.16E+08	1.40E-10
	σ_f	5.65E-03	9.17E+07	2.72E+05	7.80E-01	2.43E+00	1.38E+00	1.61E+08	2.72E+05	6.55E+07	7.66E-10
f_{18}	f_{Best}	5.75E-04	6.37E+08	1.89E+06	6.13E+01	4.02E+01	9.12E+00	9.25E+06	1.89E+06	4.98E+07	**1.35E-32**
	f_{Worst}	5.15E-02	1.21E+09	2.22E+07	5.88E+02	3.17E+02	6.09E+01	4.57E+08	2.22E+07	5.70E+08	3.71E-12
	$\bar{f}$	4.43E-03	9.42E+08	7.18E+06	1.27E+02	8.65E+01	2.39E+01	1.89E+08	7.18E+06	2.48E+08	1.49E-13
	σ_f	9.47E-03	1.64E+08	4.60E+06	9.95E+01	4.93E+01	9.50E+00	1.80E+08	4.60E+06	1.23E+08	6.80E-13
f_{19}	f_{Best}	3.82E-02	3.53E+12	1.44E+04	1.00E+10	7.59E+01	1.33E-03	7.40E+04	1.44E+04	2.14E+02	**2.00E-104**
	f_{Worst}	1.83E-01	1.12E+20	6.63E+04	1.00E+10	4.75E+02	4.37E+03	2.32E+05	6.63E+04	1.23E+04	2.88E+00
	$\bar{f}$	7.61E-02	9.05E+18	2.20E+04	1.00E+10	2.14E+02	2.01E+02	1.37E+05	2.20E+04	4.57E+03	9.60E-02
	σ_f	2.84E-02	2.48E+19	9.22E+03	0.00E+00	9.09E+01	8.16E+02	4.78E+04	9.22E+03	3.31E+03	5.25E-01
f_{20}	f_{Best}	1.47E+02	3.06E+03	1.27E+03	5.34E+02	3.25E+02	2.86E+02	9.81E+02	1.27E+03	1.41E+02	**9.90E+01**
	f_{Worst}	2.12E+02	4.18E+03	1.99E+03	6.61E+02	5.18E+02	4.96E+02	3.04E+03	1.99E+03	9.59E+02	9.90E+01
	$\bar{f}$	1.81E+02	3.66E+03	1.44E+03	6.01E+02	4.03E+02	3.81E+02	1.86E+03	1.44E+03	5.03E+02	9.90E+01
	σ_f	1.70E+01	2.56E+02	1.39E+02	2.73E+01	4.79E+01	5.37E+01	5.34E+02	1.39E+02	2.06E+02	4.40E-04
f_{21}	f_{Best}	2.81E+03	7.38E+08	2.03E+06	2.41E+03	6.78E+03	1.49E+03	6.63E+06	2.03E+06	3.61E+07	**1.09E+02**
	f_{Worst}	5.74E+03	1.33E+09	1.83E+07	7.10E+03	1.16E+05	4.40E+03	1.27E+09	1.83E+07	7.69E+08	3.10E+03
	$\bar{f}$	3.86E+03	1.00E+09	6.53E+06	3.78E+03	1.72E+04	2.14E+03	2.25E+08	6.53E+06	2.56E+08	3.92E+02
	σ_f	6.81E+02	1.48E+08	4.25E+06	8.91E+02	1.97E+04	6.70E+02	3.10E+08	4.25E+06	1.47E+08	7.86E+02
f_{22}	f_{Best}	9.95E+01	5.62E+09	1.43E+03	1.00E+10	2.34E+02	2.99E+02	1.88E+03	1.43E+03	1.26E+02	**9.90E+01**
	f_{Worst}	1.54E+02	8.00E+18	2.92E+03	1.00E+10	4.27E+02	5.73E+02	7.43E+03	2.92E+03	1.02E+03	9.90E+01
	$\bar{f}$	1.13E+02	3.12E+17	1.70E+03	1.00E+10	3.15E+02	4.38E+02	4.48E+03	1.70E+03	3.66E+02	9.90E+01
	σ_f	1.40E+01	1.47E+18	3.07E+02	0.00E+00	4.54E+01	6.27E+01	1.23E+03	3.07E+02	2.31E+02	4.02E-03
f_{23}	f_{Best}	−2.95E+02	1.82E+10	1.62E+04	1.00E+10	−1.24E+02	−2.87E+02	2.66E+08	1.62E+04	4.90E+05	**−2.98E+02**
	f_{Worst}	−2.91E+02	7.53E+20	2.23E+06	1.00E+10	1.37E+03	3.07E+03	6.40E+09	2.23E+06	1.43E+08	−2.87E+02
	$\bar{f}$	−2.94E+02	3.22E+19	4.53E+05	1.00E+10	5.15E+02	−4.16E+00	2.81E+09	4.53E+05	2.67E+07	−2.97E+02
	σ_f	7.66E-01	1.38E+20	5.90E+05	0.00E+00	4.13E+02	6.15E+02	1.52E+09	5.90E+05	3.13E+07	1.88E+00

achieves similar results than some metaheuristic algorithms. The remarkable performance of the proposed method is based on its capability of changing the optimization process between exploration and exploitation stages. Traditionally, the metaheuristic operators of metaheuristic optimization algorithms are designed to start performing exploration in the search space; then, at the final stage of the optimization process, the exploitation mechanism is performed. In the proposed mechanism, a changing frequency is added to perform both exploration and exploitation during the entire optimization process. This mechanism improves the search strategy towards the global optimum by changing the metaheuristic stages. By this changing scheme, the proposed method is capable of obtaining the best results for most of the benchmark functions evaluating a 30-dimensional search space.

To test the scalability of the proposed method in higher-dimensional search spaces, a performance comparison among EA−HC and the rest of tested metaheuristic methods considering a 100-dimensional search space is conducted. Table 3 reports the numerical results for this test. According to the table, it can be deduced that the metaheuristic operators of the proposed approach outperform the rest of its competitors evaluating at a higher-dimensional search space. The structure of the metaheuristic operators of EA−HC, conducts the search strategy into a more efficient mechanism than the rest of the algorithms evaluating most of the benchmark functions in Table A1. Some exceptions to this include the functions f_8, f_9 and f_{13}. For functions f_8 and f_9, the JADE algorithm performs quite similar to the proposed methodology. For function f_{13}, only the ABC method produces a similar objective function value than EA−HC. Based on the mean and standard deviation metrics, the JADE and ABC methods produce more consistent results than EA−HC. This may indicate that the proposed method produces less consistent results. However, the main reason for this phenomenon is based on the changing behavior of the proposed method. EA−HC method uses the execution of the exploration and exploitation operators during the entire optimization process in counterpart to most of the metaheuristic methodologies, which incorporate an exploration and exploitation stages in a fixed period of time in the optimization process. The main advantage of EA−HC over the rest of the tested algorithms is achieved by the balance of the evolutionary stages over the entire optimization process. As a principle, EA−HC considers a changing scheme among exploration and exploitation every 50 iterations; this mechanism allows EA−HC to produce a balance among evolutionary stages that conducts the search strategy by maintaining the population diversity.

Additionally, to the previous numerical experiment, to incorporate higher dimensionality in the optimization process, Table 4 reports the performance results considering 200-dimensional search space ($n = 200$).

The purpose of this experiment is to explore the capabilities of the proposed metaheuristic operators considering the optimization process

Table 4
Minimization results of benchmark functions of Table A1 with $n = 200$.

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
f_1	f_{Best}	1.51E+02	2.36E+05	3.02E+04	2.41E+03	7.80E+03	1.96E+00	1.31E+05	2.82E+02	9.12E+03	**1.51E-42**
	f_{Worst}	4.70E+03	2.99E+05	5.36E+04	3.96E+03	4.89E+04	6.16E+01	2.14E+05	4.40E+02	6.91E+04	2.69E+03
	$\bar{f}$	1.89E+03	2.73E+05	4.01E+04	3.05E+03	1.68E+04	1.23E+01	1.67E+05	3.64E+02	3.12E+04	9.34E+01
	σ_f	1.37E+03	1.56E+04	5.83E+03	3.44E+02	7.73E+03	1.59E+01	2.44E+04	3.49E+01	1.48E+04	4.91E+02
f_2	f_{Best}	3.29E+00	6.17E+39	2.15E+02	1.00E+10	5.75E+01	2.48E+00	4.26E+02	1.23E+26	2.43E+00	**1.60E-20**
	f_{Worst}	7.72E+00	3.16E+56	4.40E+02	1.00E+10	1.60E+02	1.93E+02	6.63E+02	6.63E+64	2.92E+01	2.71E+02
	$\bar{f}$	4.70E+00	1.14E+55	2.84E+02	1.00E+10	9.75E+01	2.35E+01	5.36E+02	2.59E+63	1.15E+01	3.52E+01
	σ_f	9.73E-01	5.75E+55	6.13E+01	0.00E+00	2.13E+01	4.14E+01	5.66E+01	1.22E+64	7.66E+00	7.23E+01
f_3	f_{Best}	5.53E+05	1.57E+06	1.56E+05	1.82E+05	1.55E+05	7.51E+05	3.81E+05	1.75E+05	4.56E+05	**2.64E-16**
	f_{Worst}	8.23E+05	2.70E+06	8.99E+05	3.11E+05	4.36E+05	1.36E+06	1.05E+06	2.34E+05	1.27E+06	2.14E+06
	$\bar{f}$	6.91E+05	2.17E+06	3.44E+05	2.44E+05	2.91E+05	1.06E+06	6.70E+05	2.13E+05	8.20E+05	1.11E+06
	σ_f	6.56E+04	3.25E+05	1.44E+05	3.28E+04	7.30E+04	1.55E+05	1.82E+05	1.49E+04	1.67E+05	6.47E+05
f_4	f_{Best}	9.47E+01	8.76E+01	3.29E+01	1.99E+01	4.85E+01	9.21E+01	9.41E+01	6.82E+01	9.31E+01	**8.90E-26**
	f_{Worst}	9.69E+01	9.38E+01	5.18E+01	2.86E+01	7.30E+01	9.76E+01	9.79E+01	8.17E+01	9.69E+01	1.25E+01
	$\bar{f}$	9.60E+01	9.18E+01	4.18E+01	2.29E+01	5.96E+01	9.57E+01	9.65E+01	7.56E+01	9.52E+01	4.16E-01
	σ_f	6.40E-01	1.40E+00	4.17E+00	1.86E+00	6.75E+00	1.29E+00	9.68E-01	4.04E+00	1.07E+00	2.28E+00
f_5	f_{Best}	4.29E+02	2.97E+03	9.81E+03	2.18E+02	3.55E+02	1.93E+02	1.68E+03	1.97E+02	4.80E+02	**1.77E+00**
	f_{Worst}	6.97E+02	4.27E+03	1.61E+04	2.44E+02	8.29E+02	1.98E+02	2.75E+03	3.20E+02	1.82E+03	5.00E+02
	$\bar{f}$	5.25E+02	3.78E+03	1.29E+04	2.31E+02	5.04E+02	1.95E+02	2.13E+03	2.23E+02	1.05E+03	1.56E+02
	σ_f	6.77E+01	2.68E+02	1.34E+03	4.80E+00	1.02E+02	1.42E+00	2.99E+02	3.42E+01	3.33E+02	1.21E+02
f_6	f_{Best}	1.81E+02	2.46E+05	3.05E+04	3.57E+03	7.35E+03	1.22E+02	1.18E+05	6.80E+02	1.14E+04	**0.00E+00**
	f_{Worst}	6.26E+03	3.16E+05	5.20E+04	6.67E+03	4.07E+04	1.53E+03	2.17E+05	1.41E+03	5.35E+04	4.99E+03
	$\bar{f}$	2.49E+03	2.73E+05	4.18E+04	4.42E+03	1.50E+04	5.39E+02	1.59E+05	9.81E+02	3.05E+04	2.22E+02
	σ_f	1.64E+03	1.61E+04	6.09E+03	6.20E+02	6.74E+03	3.90E+02	2.53E+04	1.89E+02	1.33E+04	9.51E+02
f_7	f_{Best}	6.22E+00	1.86E+03	2.31E+03	2.33E+00	7.49E+00	2.33E+00	9.96E+02	8.63E-01	3.94E+02	**5.88E-06**
	f_{Worst}	2.02E+01	2.96E+03	4.31E+03	4.64E+00	6.10E+01	6.15E+00	2.59E+03	1.67E+00	1.22E+03	3.53E+01
	$\bar{f}$	1.06E+01	2.36E+03	3.25E+03	3.15E+00	2.67E+01	3.36E+00	1.57E+03	1.38E+00	8.34E+02	1.18E+00
	σ_f	3.58E+00	2.54E+02	4.79E+02	5.53E-01	1.12E+01	8.44E-01	4.37E+02	1.99E-01	2.18E+02	6.44E+00
f_8	f_{Best}	6.29E+00	1.41E+02	7.21E+01	3.16E+01	9.05E+00	**2.01E+00**	2.04E+00	6.42E+00	2.09E+00	**2.01E+00**
	f_{Worst}	1.45E+01	1.51E+02	9.30E+01	4.37E+01	1.95E+01	2.08E+00	3.60E+01	1.11E+01	2.17E+00	1.65E+02
	$\bar{f}$	1.07E+01	1.46E+02	8.24E+01	3.93E+01	1.36E+01	2.05E+00	1.07E+01	8.81E+00	2.12E+00	4.55E+01
	σ_f	2.20E+00	2.34E+00	4.89E+00	2.66E+00	2.27E+00	1.70E-02	6.57E+00	1.36E+00	1.80E-02	4.98E+01
f_9	f_{Best}	1.23E+00	1.33E+02	9.92E+01	7.33E+00	4.66E+00	1.01E-02	8.09E+00	8.03E-02	7.37E-02	**1.01E-02**
	f_{Worst}	7.43E+00	1.62E+02	1.22E+02	1.37E+01	1.05E+01	1.23E-02	6.40E+01	1.22E-01	1.78E-01	1.14E+02
	$\bar{f}$	3.65E+00	1.48E+02	1.11E+02	1.07E+01	7.36E+00	1.04E-02	2.99E+01	1.00E-01	1.25E-01	2.95E+01
	σ_f	1.61E+00	6.42E+00	5.40E+00	1.56E+00	1.31E+00	4.85E-04	1.51E+01	1.13E-02	3.13E-02	3.79E+01
f_{10}	f_{Best}	4.05E+02	2.51E+03	1.69E+03	1.07E+03	6.68E+02	7.91E+02	1.68E+03	1.38E+03	1.48E+02	**0.00E+00**
	f_{Worst}	4.92E+02	2.72E+03	2.02E+03	1.38E+03	1.01E+03	1.03E+03	2.13E+03	1.78E+03	9.73E+02	1.75E+03
	$\bar{f}$	4.48E+02	2.63E+03	1.85E+03	1.23E+03	8.30E+02	9.40E+02	1.88E+03	1.60E+03	4.69E+02	4.22E+02
	σ_f	2.67E+01	5.49E+01	8.77E+01	5.80E+01	8.76E+01	5.48E+01	1.02E+02	1.09E+02	2.05E+02	5.82E+02
f_{11}	f_{Best}	1.09E+01	1.97E+01	1.31E+01	9.68E+00	9.89E+00	1.80E+01	1.99E+01	5.64E+00	8.06E+00	**8.88E-16**
	f_{Worst}	1.37E+01	2.01E+01	1.51E+01	1.59E+01	2.00E+01	2.00E+01	2.00E+01	2.05E+01	2.08E+01	1.38E+01
	$\bar{f}$	1.24E+01	1.99E+01	1.39E+01	1.23E+01	1.56E+01	1.97E+01	1.99E+01	1.66E+01	2.00E+01	1.53E+00
	σ_f	8.09E-01	1.21E-01	4.75E-01	1.63E+00	4.29E+00	5.02E-01	1.64E-02	6.16E+00	2.82E+00	3.71E+00
f_{12}	f_{Best}	1.80E+00	2.10E+03	2.67E+02	2.43E+01	5.78E+01	3.79E-01	1.08E+03	3.62E+00	5.45E+01	**0.00E+00**
	f_{Worst}	7.08E+01	2.72E+03	4.30E+02	3.88E+01	3.11E+02	1.18E+00	1.69E+03	5.03E+00	6.42E+02	1.78E+03
	$\bar{f}$	1.98E+01	2.49E+03	3.51E+02	2.98E+01	1.27E+02	7.61E-01	1.39E+03	4.39E+00	3.10E+02	6.06E+01
	σ_f	1.98E+01	1.50E+02	4.51E+01	3.44E+00	5.16E+01	2.01E-01	1.71E+02	3.14E-01	1.45E+02	3.25E+02
f_{13}	f_{Best}	−1.91E+01	2.35E+01	4.67E+01	−1.47E+01	−1.27E+01	−1.50E+01	3.07E+00	−8.77E+00	−1.88E+01	**−2.00E+01**
	f_{Worst}	−1.69E+01	3.06E+01	6.89E+01	−8.71E+00	−7.11E+00	−1.07E+01	1.44E+01	−2.86E+00	−3.41E+00	3.94E+00
	$\bar{f}$	−1.80E+01	2.72E+01	5.45E+01	−1.23E+01	−1.02E+01	−1.32E+01	1.02E+01	−6.44E+00	−1.30E+01	−1.86E+01
	σ_f	5.82E-01	1.56E+00	4.69E+00	1.55E+00	1.56E+00	1.01E+00	2.30E+00	1.48E+00	3.78E+00	5.43E+00
f_{14}	f_{Best}	4.53E+00	2.16E+03	6.56E+01	8.00E+00	9.02E+00	3.03E+00	8.17E+02	2.73E+00	1.89E+02	**2.72E+00**
	f_{Worst}	6.74E+01	4.24E+03	1.64E+02	9.86E+00	4.73E+01	5.43E+00	2.13E+03	2.77E+00	2.21E+03	2.72E+00
	$\bar{f}$	2.13E+01	3.22E+03	1.11E+02	8.84E+00	2.21E+01	4.03E+00	1.40E+03	2.73E+00	9.68E+02	2.72E+00
	σ_f	1.52E+01	4.49E+02	2.50E+01	4.77E-01	9.36E+00	6.60E-01	3.33E+02	8.46E-03	4.83E+02	2.11E-03
f_{15}	f_{Best}	1.47E-01	9.94E+01	5.43E+01	8.75E-01	3.09E+00	5.73E-04	4.35E+01	1.27E-01	1.69E+00	**−1.01E+00**
	f_{Worst}	4.13E+00	1.20E+02	7.83E+01	1.55E+00	1.05E+01	2.50E-02	7.79E+01	1.78E-01	2.74E+01	1.63E+01
	$\bar{f}$	1.09E+00	1.10E+02	6.61E+01	1.23E+00	5.46E+00	4.42E-03	6.31E+01	1.53E-01	1.22E+01	1.77E+00

(*continued on next page*)

Table 4 (*continued*)

		ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
	σ_f	9.29E-01	3.90E+00	5.96E+00	1.44E-01	1.84E+00	5.22E-03	7.47E+00	1.47E-02	7.11E+00	4.21E+00
f_{16}	f_{Best}	−6.29E+04	−1.38E+04	−1.33E+04	−3.60E+04	−5.02E+04	−4.22E+04	−4.96E+04	−4.98E+04	−1.20E+04	**−8.38E+04**
	f_{Worst}	−5.70E+04	−9.91E+03	−8.02E+03	−3.35E+04	−4.16E+04	−3.70E+04	−3.52E+04	−4.06E+04	−9.80E+03	−9.37E+03
	$\bar{f}$	−6.00E+04	−1.13E+04	−1.08E+04	−3.47E+04	−4.49E+04	−3.86E+04	−4.14E+04	−4.54E+04	−1.05E+04	−2.92E+04
	σ_f	1.44E+03	1.08E+03	1.53E+03	7.34E+02	2.29E+03	1.59E+03	3.65E+03	2.37E+03	5.53E+02	2.68E+04
f_{17}	f_{Best}	5.73E-02	1.31E+09	6.86E+05	8.60E+00	1.83E+04	1.24E+01	4.68E+08	2.23E+01	3.97E+08	**9.85E-18**
	f_{Worst}	1.17E+06	1.93E+09	8.52E+06	1.79E+01	2.89E+06	2.94E+01	1.72E+09	4.16E+01	1.68E+09	5.88E+06
	$\bar{f}$	5.38E+04	1.59E+09	2.87E+06	1.17E+01	7.54E+05	2.06E+01	1.05E+09	3.28E+01	9.42E+08	1.96E+05
	σ_f	2.24E+05	1.64E+08	1.93E+06	1.73E+00	8.48E+05	4.93E+00	2.94E+08	5.74E+00	3.13E+08	1.07E+06
f_{18}	f_{Best}	3.41E+00	2.59E+09	6.39E+06	2.15E+03	1.28E+06	2.46E+02	8.94E+08	2.93E+02	4.59E+08	**7.43E-15**
	f_{Worst}	1.06E+05	3.84E+09	5.37E+07	5.07E+04	1.08E+08	3.90E+02	3.03E+09	4.24E+02	2.75E+09	1.96E+07
	$\bar{f}$	8.25E+03	3.30E+09	2.95E+07	1.23E+04	1.21E+07	3.05E+02	1.90E+09	3.70E+02	1.60E+09	1.09E+06
	σ_f	2.19E+04	3.06E+08	1.12E+07	1.18E+04	1.99E+07	3.84E+01	5.02E+08	2.91E+01	5.11E+08	4.23E+06
f_{19}	f_{Best}	1.63E+01	7.42E+43	2.68E+04	1.00E+10	9.36E+03	2.92E+00	2.64E+05	4.44E+22	2.49E+03	**9.68E-24**
	f_{Worst}	8.06E+01	2.64E+58	1.23E+05	1.00E+10	2.35E+04	8.79E+02	5.68E+05	4.68E+69	5.93E+04	2.60E+05
	$\bar{f}$	3.41E+01	1.16E+57	5.64E+04	1.00E+10	1.49E+04	8.09E+01	3.79E+05	1.56E+68	2.16E+04	1.02E+04
	σ_f	1.35E+01	5.01E+57	2.26E+04	0.00E+00	3.52E+03	2.18E+02	5.70E+04	8.55E+68	1.07E+04	4.74E+04
f_{20}	f_{Best}	1.63E+03	8.44E+03	2.68E+03	1.44E+03	1.47E+03	9.74E+02	5.10E+03	1.84E+03	9.99E+02	**1.99E+02**
	f_{Worst}	1.32E+03	1.01E+04	3.38E+03	1.70E+03	2.41E+03	1.51E+03	7.91E+03	2.55E+03	3.37E+03	4.03E+03
	$\bar{f}$	6.39E+02	9.18E+03	3.13E+03	1.58E+03	1.89E+03	1.21E+03	6.92E+03	2.18E+03	2.14E+03	7.02E+02
	σ_f	5.50E+02	4.51E+02	1.58E+02	5.46E+01	2.17E+02	1.33E+02	6.32E+02	1.87E+02	6.64E+02	1.16E+03
f_{21}	f_{Best}	2.05E+04	2.47E+09	8.23E+06	2.80E+04	1.79E+06	1.02E+04	6.35E+08	1.24E+04	4.67E+08	**2.19E+02**
	f_{Worst}	2.54E+07	3.66E+09	5.18E+07	2.01E+05	3.15E+07	4.93E+04	2.89E+09	2.39E+04	2.22E+09	3.18E+08
	$\bar{f}$	1.63E+06	3.25E+09	2.99E+07	6.44E+04	6.99E+06	1.61E+04	1.98E+09	1.55E+04	1.52E+09	2.32E+07
	σ_f	5.31E+06	3.20E+08	9.91E+06	3.33E+04	6.07E+06	9.04E+03	6.10E+08	2.83E+03	3.95E+08	6.89E+07
f_{22}	f_{Best}	2.96E+02	3.73E+40	2.95E+03	1.00E+10	1.47E+03	1.04E+03	9.54E+03	5.99E+23	8.28E+02	**1.99E+02**
	f_{Worst}	5.02E+02	4.55E+54	5.48E+03	1.00E+10	2.27E+03	1.87E+03	1.86E+04	8.90E+65	2.47E+03	3.61E+03
	$\bar{f}$	3.75E+02	3.25E+53	3.72E+03	1.00E+10	1.82E+03	1.37E+03	1.31E+04	2.97E+64	1.35E+03	7.32E+02
	σ_f	4.78E+01	8.78E+53	5.79E+02	0.00E+00	2.10E+02	2.12E+02	1.91E+03	1.63E+65	4.33E+02	1.07E+03
f_{23}	f_{Best}	2.96E+02	3.91E+45	7.11E+05	1.00E+10	1.81E+04	9.98E+02	5.24E+09	8.19E+09	1.48E+07	**−6.03E+02**
	f_{Worst}	5.02E+02	8.97E+59	1.32E+18	1.00E+10	2.06E+07	5.28E+07	1.05E+10	3.72E+73	4.52E+08	3.90E+06
	$\bar{f}$	3.75E+02	3.23E+58	4.41E+16	1.00E+10	1.05E+06	1.78E+06	7.01E+09	1.24E+72	2.22E+08	1.31E+05
	σ_f	4.78E+01	1.64E+59	2.41E+17	0.00E+00	3.77E+06	9.64E+06	1.24E+09	6.80E+72	1.12E+08	7.11E+05

within a higher dimensional search space. As it can be demonstrated, Table 4 indicates that the proposed EA−HC method outperforms the rest of the evolutionary methodologies considered in the numerical simulation for most of the benchmark functions. The only exception to this considers the case of the JADE method in function f_8. Where the EA−HC and JADE produce the same fitness value.

According to the numerical results to compare the numerical results of the evolutionary methods considered, it is evident that the proposed method presents a remarkable performance in higher-dimensional search spaces. The structure of the evolutionary mechanism of EA−HC, produces better results than its competitors. The numerical results in higher dimensional search spaces suggest that the changing character of EA−HC presents a higher level of scalability since it outperforms the rest of the tested algorithms in the majority of benchmark functions.

To statistically corroborate the numerical results from Tables 2–4, a non-parametric test is conducted. In this study, the Wilcoxon rank-sum test [63] has been conducted in order to validate the performance results. The Wilcoxon test has been applied considering the 0.05 significance value over the 30 independent runs for each benchmark function. Table 5 reports the p-values considering the performance results of Table 2 (where $n = 30$) obtained by the Wilcoxon test. Additionally, Table 6 reports the p-values considering the performance results of Table 3 (where $n = 100$) obtained by the rank-sum test. Also, Table 7 presents the rank-sum test values of Table 4 (where $n = 200$). In Tables 5–7, a pair-wise comparison among EA—HC and the rest of the tested algorithms is presented. Under such circumstances, nine groups are considered in the study; EA—HC vs. ABC, EA—HC vs. DE, EA—HC vs. PSO, EA—HC vs. CS, EA—HC vs. DS, EA—HC vs. JADE, EA—HC vs. MFO, EA—HC vs. MVO and EA—HC vs. SCA.

In the statistical experiment, it is considered that there is no significant difference in a group (null hypothesis H_0). Also, it is considered as an alternative hypothesis H_1 that there is a significant difference in a group. To make a clear visualization of the results, Tables 5–7, adopt the symbols▲ ▼, and ▶. The symbol ▲, represents that EA—HC achieves significantly better results than a given competitor. The symbol ▼, represents that EA—HC produces worse results than its competitor, and the symbol ▶ represents the situation when the Wilcoxon test is not able to distinguish between the numerical results. According to the p-values from Table 5($n = 30$), it is demonstrated that for function f_6 the groups EA—HC vs. ABC, EA—HC vs. CS, EA—HC vs. DS, EA—HC vs. JADE, EA—HC vs. MFO and EA—HC vs. SCA the rank-sum is not able to distinguish among the results. For function f_8, the groups EA—HC vs. ABC, EA—HC vs. CS, EA—HC vs. DS, EA—HC vs. JADE, EA—HC vs. MVO clearly indicate a similar performance of these technique comparing with the proposed approach. According to the Wilcoxon test,

Table 5
p-values produced by Wilcoxon rank sum test comparing EA−HC vs. ABC, EA−HC vs. DE, EA−HC vs. PSO, EA−HC vs. CS, EA−HC vs. DS, EA−HC vs. JADE, EA−HC vs. MFO, EA−HC vs. MVO and EA−HC vs. SCA over the averaged fitness value $\bar{f}$ for each function from Table 2.

EA-HC vs.	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA
f_1	1.066E-07▲	1.558E-08▲	2.249E-07▲	6.765E-05▲	1.491E-06▲	2.028E-07▲	8.476E-05▲	6.765E-05▲	1.635E-05▲
f_2	9.958E-05▲	3.019E-11▲	3.019E-11▲	3.010E-07▲	3.037E-06▲	4.713E-04▲	3.061E-11▲	1.193E-06▲	6.463E-10▲
f_3	6.184E-10▲	3.019E-11▲	5.462E-06▲	3.019E-11▲	3.019E-11▲	3.196E-09▲	5.183E-10▲	3.019E-11▲	7.380E-10▲
f_4	6.045E-07▲	3.019E-11▲	1.173E-09▲	3.019E-11▲	3.019E-11▲	8.841E-07▲	2.534E-12▲	3.019E-11▲	3.338E-11▲
f_5	3.019E-11▲	3.019E-11▲	3.689E-11▲	3.019E-11▲	6.695E-11▲	3.019E-11▲	1.357E-07▲	1.613E-10▲	3.019E-11▲
f_6	0.0000E00▶	4.645E-08▲	2.701E-03▲	0.0000E00▶	0.0000E00▶	0.0000E00▶	0.0000E00▶	1.825E-04▲	0.0000E00▶
f_7	3.019E-11▲	6.065E-11▲	7.043E-07▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	1.374E-05▲	3.019E-11▲	3.019E-11▲
f_8	0.0000E00▶	2.388E-04▲	5.561E-04▲	0.0000E00▶	0.0000E00▶	0.0000E00▶	9.424E-14▲	0.0000E00▶	3.338E-11▲
f_9	0.0000E00▶	3.019E-11▲	3.019E-11▲	0.0000E00▶	0.0000E00▶	0.0000E00▶	9.434E-14▲	3.019E-11▲	3.019E-11▲
f_{10}	4.975E-11▲	6.518E-09▲	5.434E-08▲	3.279E-02▲	3.497E-09▲	1.698E-08▲	4.342E-05▲	7.341E-07▲	7.695E-08▲
f_{11}	6.749E-05▲	1.422E-08▲	1.335E-02▲	3.980E-04▲	6.749E-05▲	5.394E-03▲	4.262E-04▲	3.980E-06▲	3.559E-06▲
f_{12}	4.998E-09▲	1.473E-07▲	1.127E-05▲	1.107E-06▲	1.156E-07▲	2.601E-08▲	1.199E-05▲	1.107E-06▲	4.112E-07▲
f_{13}	0.0000E00▶	2.262E-07▲	2.504E-11▲	3.019E-11▲	0.0000E00▶	0.0000E00▶	0.0000E00▶	1.042E-07▲	0.0000E00▶
f_{14}	3.019E-11▲	1.776E-10▲	4.077E-11▲	3.019E-11▲	3.019E-11▲	3.016E-11▲	1.964E-11▲	3.019E-11▲	3.019E-11▲
f_{15}	5.572E-10▲	1.010E-08▲	6.876E-09▲	1.066E-07▲	8.484E-09▲	8.891E-10▲	5.139E-06▲	1.156E-07▲	1.429E-08▲
f_{16}	0.0000E00▶	3.231E-05▲	6.736E-06▲	3.019E-11▲	0.0000E00▶	3.019E-11▲	9.434E-14▲	4.077E-11▲	1.359E-07▲
f_{17}	3.019E-11▲	3.019E-11▲	6.121E-10▲	3.158E-10▲	3.019E-11▲	3.019E-11▲	1.594E-13▲	1.205E-10▲	1.205E-10▲
f_{18}	3.019E-11▲	8.152E-11▲	5.572E-10▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	1.017E-13▲	3.019E-11▲	1.205E-11▲
f_{19}	1.107E-06▲	2.227E-09▲	1.621E-05▲	6.765E-05▲	1.107E-06▲	2.491E-06▲	4.913E-05▲	6.765E-05▲	1.491E-06▲
f_{20}	0.0000E00▶	2.154E-10▲	5.497E-06▲	8.484E-09▲	0.0000E00▶	3.019E-11▲	4.670E-09▲	7.690E-08▲	0.0000E00▶
f_{21}	3.019E-11▲	3.019E-11▲	3.689E-11▲	3.019E-11▲	3.019E-11▲	0.0000E00▶	8.649E-13▲	3.019E-11▲	3.019E-11▲
f_{22}	0.0000E00▶	5.490E-11▲	3.874E-03▲	2.596E-05▲	0.0000E00▶	1.107E-06▲	2.564E-07▲	3.156E-05▲	0.0000E00▶
f_{23}	3.019E-11▲	3.019E-11▲	5.607E-05▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	9.986E-09▲	3.019E-11▲	6.722E-10▲
▲	16	23	23	20	16	18	21	22	19
▼	0	0	0	0	0	0	0	0	0
▶	7	0	0	3	7	5	2	1	4

Table 6
p-values produced by Wilcoxon rank sum test comparing EA−HC vs. ABC, EA−HC vs. DE, EA−HC vs. PSO, EA−HC vs. CS, EA−HC vs. DS, EA−HC vs. JADE, EA−HC vs. MFO, EA−HC vs. MVO and EA−HC vs. SCA over the averaged fitness value $\bar{f}$ for each function from Table 3.

EA-HC vs.	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA
f_1	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	2.194E-08▲	3.019E-11▲	3.019E-11▲
f_2	1.066E-07▲	3.019E-11▲	2.572E-07▲	1.211E-12▲	2.530E-04▲	4.744E-06▲	5.461E-09▲	3.019E-11▲	6.526E-07▲
f_3	3.081E-08▲	3.019E-11▲	1.193E-06▲	3.019E-11▲	3.645E-08▲	7.695E-08▲	1.681E-04▲	3.019E-11▲	8.120E-04▲
f_4	3.019E-11▲	3.019E-11▲	6.282E-06▲	3.019E-11▲	3.019E-11▲	5.572E-10▲	3.019E-11▲	6.736E-06▲	3.019E-11▲
f_5	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	7.233E-09▲	3.019E-11▲	3.019E-11▲
f_6	2.982E-11▲	3.019E-11▲	3.019E-11▲	3.014E-11▲	3.019E-11▲	2.993E-11▲	1.354E-07▲	3.010E-11▲	3.019E-11▲
f_7	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	6.207E-04▲	3.019E-11▲	4.329E-05▲
f_8	3.019E-11▲	2.138E-06▲	8.352E-08▲	3.019E-11▲	3.019E-11▲	0.0000E00▶	3.099E-11▲	3.019E-11▲	3.019E-11▲
f_9	3.019E-11▲	2.028E-07▲	4.185E-09▲	3.019E-11▲	3.019E-11▲	0.0000E00▶	3.019E-11▲	3.019E-11▲	3.019E-11▲
f_{10}	3.019E-11▲	3.019E-11▲	7.064E-09▲	3.624E-11▲	4.440E-07▲	8.165E-05▲	9.524E-08▲	4.186E-05▲	6.526E-07▲
f_{11}	5.572E-10▲	3.019E-11▲	6.120E-10▲	8.484E-09▲	7.772E-09▲	3.820E-10▲	3.019E-11▲	8.841E-07▲	6.736E-06▲
f_{12}	3.019E-11▲	3.019E-11▲	2.371E-10▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	2.159E-07▲	3.019E-11▲	3.338E-11▲
f_{13}	0.0000E00▶	3.019E-11▲	3.019E-11▲	5.572E-10▲	3.822E-10▲	5.572E-10▲	3.099E-06▲	2.227E-09▲	9.918E-11▲
f_{14}	3.019E-11▲	3.019E-11▲	5.494E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	1.243E-05▲	3.099E-11▲	1.529E-05▲
f_{15}	5.572E-10▲	3.019E-11▲	3.830E-05▲	5.572E-10▲	5.572E-10▲	3.3384-11▲	1.018E-05▲	5.572E-10▲	8.810E-10▲
f_{16}	3.099E-11▲	1.105E-04▲	1.047E-04▲	3.099E-11▲	3.099E-11▲	3.019E-11▲	3.099E-11▲	3.019E-11▲	4.182E-09▲
f_{17}	3.099E-11▲	3.019E-11▲	2.015E-08▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	4.981E-04▲	3.019E-11▲	3.019E-11▲
f_{18}	3.019E-11▲	3.019E-11▲	3.689E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	2.342E-10▲	3.019E-11▲	2.609E-10▲
f_{19}	3.019E-11▲	3.019E-11▲	2.194E-08▲	1.211E-12▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲
f_{20}	3.019E-11▲	3.019E-11▲	2.665E-09▲	3.099E-11▲	3.019E-11▲	3.019E-11▲	1.356E-06▲	3.384E-11▲	3.099E-11▲
f_{21}	3.019E-11▲	3.019E-11▲	1.270E-09▲	3.019E-11▲	3.199E-11▲	3.019E-11▲	9.753E-05▲	3.019E-11▲	8.114E-10▲
f_{22}	3.019E-11▲	3.019E-11▲	5.835 E-11▲	1.218E-12▲	3.820E-10▲	5.572E-10▲	7.391E-11▲	3.019E-11▲	1.632E-10▲
f_{23}	3.019E-11▲	3.019E-11▲	1.328E-10▲	1.218E-12▲	3.019E-11▲	3.099E-11▲	3.019E-11▲	5.527E-10▲	1.846E-06▲
▲	22	23	23	23	23	21	23	23	23
▼	0	0	0	0	0	0	0	0	0
▶	1	0	0	0	0	2	0	0	0

Table 7
p-values produced by Wilcoxon rank-sum test comparing EA−HC vs. ABC, EA−HC vs. DE, EA−HC vs. PSO, EA−HC vs. CS, EA−HC vs. DS, EA−HC vs. JADE, EA−HC vs. MFO, EA−HC vs. MVO and EA−HC vs. SCA over the averaged fitness value $\bar{f}$ for each function from Table 4.

EA-HC vs.	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA
f_1	2.371E-10▲	3.099E-11▲	3.099E-11▲	4.503E-11▲	3.019E-11▲	2.605E-08▲	3.019E-11▲	5.577E-10▲	3.019E-11▲
f_2	2.499E-03▲	3.019E-11▲	3.472E-10▲	1.218E-12▲	2.678E-06▲	1.056E-03▲	3.019E-11▲	3.019E-11▲	2.380E-03▲
f_3	1.442E-02▲	6.720E-10▲	3.157E-05▲	4.118E-06▲	1.167E-05▲	1.335E-03▲	6.372E-03▲	1.498E-06▲	3.031E-02▲
f_4	3.019E-11▲	3.099E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲	3.019E-11▲
f_5	9.916E-11▲	3.019E-11▲	3.019E-11▲	9.519E-06▲	1.3289E-10▲	5.9969E-02▲	3.0199E-11▲	3.988E-04▲	3.334E-11▲
f_6	1.871E-10▲	2.365E-12▲	2.365E-12▲	3.554E-11▲	2.367E-12▲	1.238E-09▲	2.365E-12▲	1.234E-09▲	2.367E-12▲
f_7	5.577E-10▲	3.019E-11▲	3.019E-11▲	5.577E-10▲	3.158E-10▲	5.572E-10▲	3.019E-11▲	5.527E-10▲	3.019E-11▲
f_8	6.627E-02▲	5.572E-10▲	1.232E-02▲	3.957E-02▲	6.843E-02▲	0.0000E00▶	6.952E-02▲	6.673E-02▲	1.877E-01▲
f_9	6.623E-02▲	3.099E-11▲	2.609E-10▲	6.200E-02▲	6.627E-02▲	5.593E-02▲	1.452E-02▲	6.623E-02▲	6.623E-01▲
f_{10}	7.694E-02▲	2.721E-11▲	4.494E-11▲	6.415E-06▲	8.789E-04▲	7.376E-05▲	4.061E-11▲	1.554E-09▲	4.625E-02▲
f_{11}	5.468E-10▲	2.954E-11▲	1.301E-10▲	3.744E-10▲	1.433E-10▲	2.924E-11▲	2.924E-11▲	6.518E-10▲	4.864E-11▲
f_{12}	1.796E-08▲	1.795E-11▲	3.528E-10▲	3.521E-10▲	3.581E-10▲	8.046E-06▲	3.581E-10▲	8.324E-07▲	3.521E-10▲
f_{13}	4.414E-09▲	1.271E-11▲	1.271E-11▲	4.44E-09▲	4.414E-09▲	4.411E-09▲	1.410E-11▲	4.414E-09▲	4.414E-09▲
f_{14}	3.023E-11▲	3.123E-11▲	3.013E-11▲	3.013E-11▲	3.012E-11▲	3.012E-11▲	3.012E-11▲	3.012E-11▲	3.013E-11▲
f_{15}	4.351E-05▲	3.099E-11▲	3.019E-11▲	6.750E-05▲	4.117E-06▲	1.875E-03▲	3.019E-11▲	6.650E-05▲	1.5581E-0▲
f_{16}	1.337E-05▲	2.385E-04▲	1.249E-05▲	3.981E-04▲	3.981E-04▲	3.981E-04▲	3.988E-04▲	3.988E-04▲	7.040E-07▲
f_{17}	4.686E-08▲	3.019E-11▲	4.197E-10▲	8.488E-09▲	5.527E-10▲	8.848E-09▲	3.099E-11▲	8.448E-09▲	3.099E-11▲
f_{18}	2.1959E-07▲	3.0199E-11▲	4.975E-11▲	8.4848E-09▲	2.9215E-09▲	8.4848E-09▲	3.019E-11▲	8.484E-09▲	3.019E-11▲
f_{19}	7.959E-03▲	3.0199E-11	5.577E-10▲	1.211E-12▲	7.186E-09▲	7.284E-03▲	3.019E-11▲	3.019E-11▲	3.849E-09▲
f_{20}	9.539E-06▲	3.019E-11▲	1.680E-08▲	9.519E-06▲	9.519E-06▲	9.539E-06▲	3.019E-11▲	7.220E-06▲	2.493E-06▲
f_{21}	8.649E-03▲	3.019E-11▲	1.286E-06▲	6.200E-02▲	9.797E-05▲	3.253E-02▲	3.019E-11▲	3.794E-02▲	3.019E-11▲
f_{22}	3.961E-04▲	2.953E-11▲	3.406E-10▲	1.177E-12▲	1.808E-05▲	7.594E-05▲	2.953E-11▲	2.953E-11▲	7.147E-05▲
f_{23}	7.487E-03▲	3.019E-11▲	1.205E-10▲	1.218E-12▲	5.072E-10▲	2.002E-06▲	3.019E-11▲	3.019E-11▲	3.099E-11▲
▲	23	23	23	23	23	22	23	23	23
▼	0	0	0	0	0	0	0	0	0
▶	0	0	0	0	0	1	0	0	0

the p-values for functions f_9 and f_{13}, suggest that EA—HC produces similar results to ABC, CS, DS, JADE, MFO and SCA. In function f_{16}, the p-value from the groups EA—HC vs. ABC and EA—HC vs. DS indicates that the rank-sum is not able to distinguish among the numerical results from both algorithms. Also, the groups EA—HC vs. ABC, EA—HC vs. DS and EA—HC vs. SCA perform similarly in functions f_{20} and f_{22}. Finally, the group EA—HC vs. JADE, statistically corroborates the results from Table 2 in function f_{21}. According to the p-values from Table 6($n = 100$), it is quite evident the superior performance of the proposed approach against each metaheuristic algorithm considered in the experimental study. As can be demonstrated, only in functions f_8 and f_9 the Wilcoxon test indicates that EA—HC and JADE perform quite similarly. Also, for function f_{13}, the ABC and EA—HC methods obtain the best fitness value than the rest of evolutionary methodologies. For the rest of the entries, the EA—HC method outperforms the rest of the competitors considering 100-dimensional search space for each benchmark functions.

Considering the p-value from Table 7($n = 200$), it is quite evident that the EA−HC method competes directly with the JADE algorithm since both algorithms produce similar outcomes. However, considering the total amount of experiment, the scalability of the evolutionary operators of EA−HC promotes a balance among evolutionary stages (exploration/exploitation) due to its changing capabilities among these stages.

From the statistical results reported in Tables 5–7, it can be deduced that the changing feature of the evolutionary structure of the proposed approach produces more robust and scalable results than the rest of metaheuristic algorithms, maintaining the population diversity and balance among exploration/exploitation over the optimization process.

5.3. Convergence

In this section, convergence analysis of the proposed approach and the tested metaheuristic algorithms is presented. The performance comparison exposed in Tables 2–4, reports the capabilities of the proposed approach in terms of fitness values. However, in most of the reported literature, a converge study must be included to evaluate the velocity which metaheuristic approaches reach during the optimization process for each benchmark function. In the convergence experiment, the convergence data was selected based on the average fitness values and considering the evaluation over 200-dimensional search spaces from Table 4.

Fig. 4 indicates that the convergence rate of the proposed method is the fastest regarding the convergence speed of the tested algorithms. As a result, the remarkable performance of EA−HC, suggests that its changing characteristic produces more reliable results by maintaining the population diversity during the entire optimization process.

5.4. Engineering design problems

Optimization problems are defined as a mathematical model to represent many real-world problems. The main purpose of optimization is the finding process of the optimal solution for a given objective function. Since many disciplines, such as engineering, medicine, economics, etc. formulate their problems in terms of minimization/maximization, recently developed metaheuristic techniques should be tested under such examples. Traditionally, to measure the performance on metaheuristic algorithms over real-world applications, several engineering design problems are evaluated as objective functions. In this study, the performance of EA−HC is tested over three common

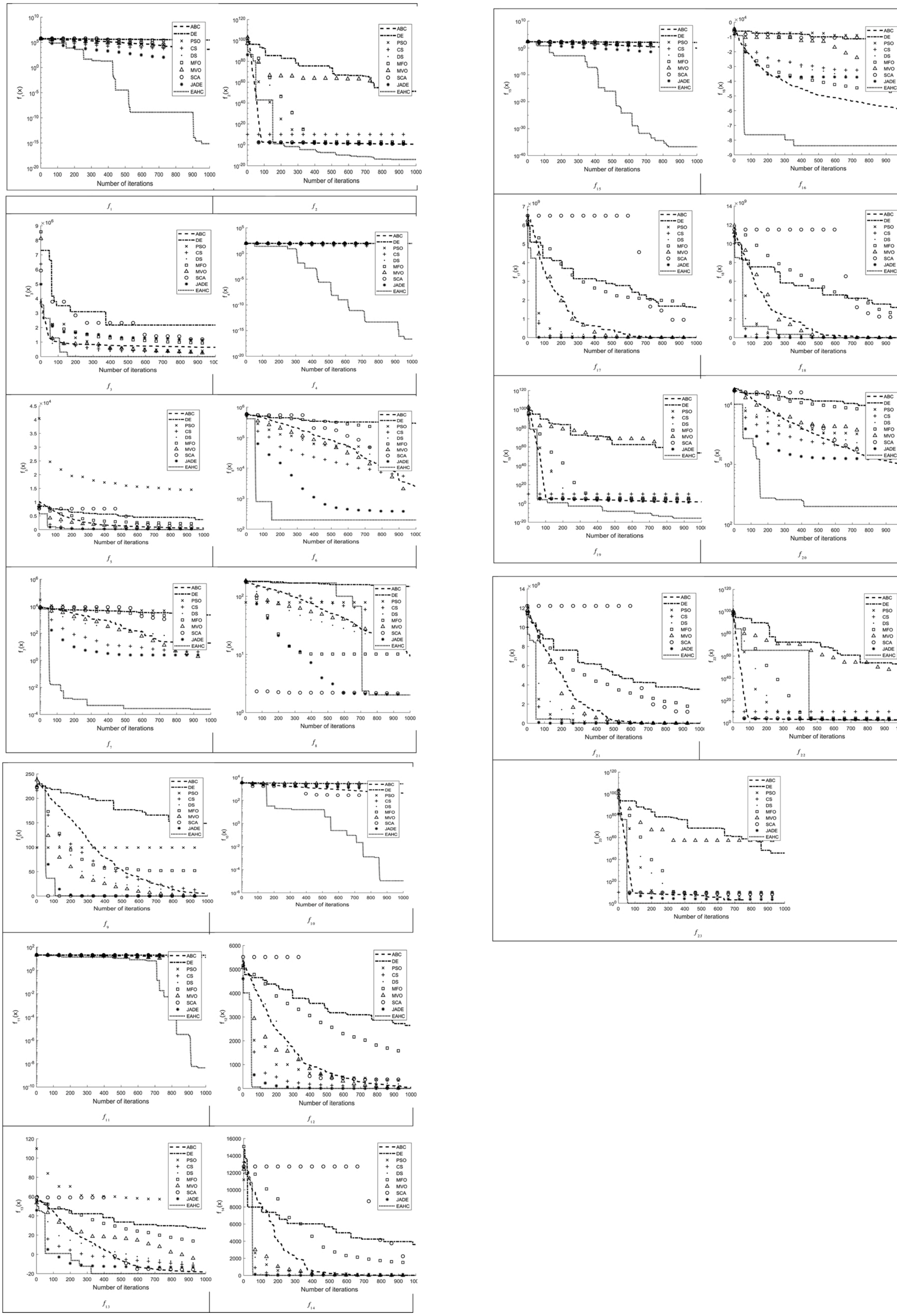

Fig. 4. Convergence graphs from Table 4.

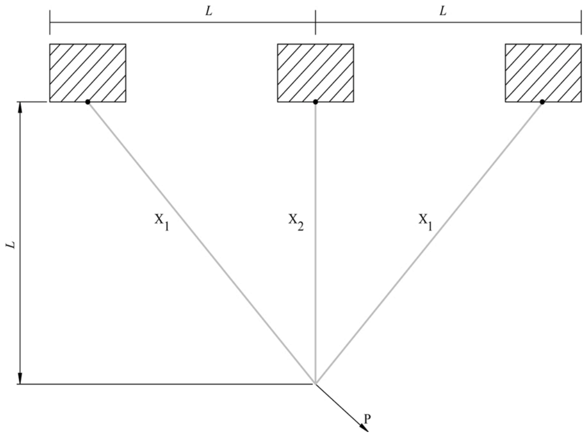

Fig. 5. Description of the three-bar truss design problem.

engineering design problems [64,65]; The three-bar truss design problem (Table B1 in Appendix B), the tension/compression spring design problem (Table B2 in Appendix B), and the welded beam design problem (Table B3 in Appendix B). The following section presents the numerical results for this test.

5.4.1. Three-bar truss design problem

The three-bar truss design optimization is conceived as a design optimization problem design. The main purpose of this task is to minimize the volume of a loaded three-bar truss, which has been subjected to several design constraints by each truss. The mathematical model of this optimization task considers a 2-dimensional search space with three inequalities. The experimental setting considers the configuration described in Section 5.1 with $gen = 1000$ for each algorithm. The graphical description of the problem is presented in Fig. 5, and the numerical results are presented in Table 8. In the table, the parameters of the optimization problem are presented. Such entries indicate the decision variables, the constraints and the fitness value.

Table 8 indicates that the proposed method is capable of obtaining competitive results than CS, DS, JADE and MVO methods. To statistically validate the results from Table 8, Table 9 reports the worst, mean, standard deviation and the best fitness values achieved by each metaheuristic technique. As it can be deduced, the proposed EA−HC achieves similar fitness value to CS, DS, JADE and MVO. However, it presents higher mean and standard deviation values due to its changing characteristics in the evolutionary structure in CS, DS and MVO.

5.4.2. Tension/compression spring design problem

The spring design optimization task presents an optimization design problem to test the ability of the metaheuristic methods by solving the minimization process of a weight tension/compression spring. The problem involves 3-dimensional search space; Wire diameter $W(x_1)$, mean coil diameter $d(x_2)$ and the number of active coils $L(x_3)$. It also presents three non-linear inequality constraints. The experimental results for this experiment are presented in Table 10. Fig. 6 presents a schematic of the tension/compression spring problem.

From Table 10, it can be shown that EA−HC produces worse fitness value than CS, DS, JADE, MFO, MVO and SCA. Even if these algorithms outperform the proposed method, the EA−HC method produces competitive results compared with the state-of-art algorithms. This effect can be produced by the efficient performance achieved by the dynamic balance among the coward and heroical evolutionary operators of EA−HC. Table 11 reports the statistical results for this experiment.

5.4.3. Welded beam design problem

The welded beam optimization design problem corresponds to a complex engineering optimization task. The main objective of this approach consists of finding the lowest cost of a welded beam. The graphical description of the design problem is given in Fig. 7. The welded beam design process involves a 4-dimensional search space: Width $h(x_1)$, length $l(x_2)$, depth $t(x_3)$ and thickness $b(x_4)$. Additionally, this optimization task consists of 7 constraints described in Table B3 in Appendix B. The numerical results are reported in Table 12.

From Table 12, it can be demonstrated the EA−HC is capable of achieving quite similar results than CS and MFO. To corroborate the experimental results from Table 12, Table 13 reports that EA−HC produces results with greater mean and standard deviation. However, this is not a limitation of the proposed approach, since it uses changing framework overexploitation and exploration stages during the entire evolutionary process.

6. Conclusions

The performance of a metaheuristic approach is affected mainly by the interactions among its individuals rather than any other factor. Although all metaheuristic schemes model interactions emulating very different processes or systems, the used operators are very similar. Such common mathematical processes have been designed without considering the final global result obtained by the individual interactions. On the other hand, agent-based systems provide a solid theory and a set of consistent models that allow characterizing global behavioral patterns produced by the collective interaction of the individuals from the set of simple rules. Under this perspective, several agent-based concepts and models that generate effective search behaviors can be used to produce or improve efficient optimization algorithms.

In this paper, a novel metaheuristic technique considering agent-

Table 9
Statistical results for the three-bar truss design problem.

Algorithm	Worst	Mean	Std	Best
ABC	279.7849	279.7372	0.0125	279.7452
DE	280.1867	279.8096	0.0902	279.7472
PSO	282.8226	93.2079	29.9639	280.5426
CS	279.7245	279.7245	1.3843e-13	279.7245
DS	280.0971	279.7484	0.0677	279.7245
JADE	282.8427	279.8500	0.5675	279.7245
MFO	280.1945	279.7763	0.1028	279.7246
MVO	279.7270	279.7251	5.7728e-04	279.7245
SCA	282.8427	281.4059	1.5624	279.7312
EA-HC	279.9114	279.7491	0.0411	279.7245

Table 8
Numerical results for the three-bar truss problem.

Parameter	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
x_1	0.8643	0.8708	0.8573	0.8698	0.8700	0.8837	0.8702	0.8697	0.8730	0.8522
x_2	0.2278	0.2149	0.2710	0.2166	0.2162	0.1945	0.2157	0.2167	0.2101	0.2528
$g_1(x)$	−1.49E-04	−3.60E-04	−0.0275	−2.2204e-16	−6.2774e-10	−4.7746e-05	0	−2.5611e-07	−5.7400e-05	0.0001
$g_2(x)$	−1.6858	−1.7029	−1.6396	−1.7005	−1.7010	−1.7295	−1.7018	−1.7005	−1.7091	−1.6532
$g_3(x)$	−0.3143	−0.2975	−0.3878	−0.2995	−0.2990	−0.2704	−0.2982	−0.2995	−0.2909	0.3467
$f(x)$	279.7452	279.7472	280.5426	279.7245	279.7245	279.7245	279.7246	279.7245	279.7312	279.7245

Table 10
Numerical results for the spring design problem.

Parameter	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
w	0.0500	0.0500	0.0516	0.0500	0.0500	0.0500	0.0500	0.0500	0.0500	0.0518
d	0.4532	0.4278	0.3560	0.4800	0.4800	0.3712	0.4800	0.4800	0.4796	0.4156
L	4.8539	5.8778	11.3295	4.0563	4.0577	14.352	4.0563	4.0580	4.0806	9.4088
$g_1(x)$	−0.0070	−0.0257	−0.000006	−1.5477e-13	−4.8364e-06	−0.6374	−0.0025	−6.6620e-05	−0.0035	−0.3079
$g_2(x)$	−0.1001	−0.1896	−0.000013	−2.0761e-14	−2.1545e-04	−0.3702	−1.6653e-15	−2.1897e-04	−0.0015	−0.2361
$g_3(x)$	−6.0440	−5.5282	−4.0523	−6.5135	−6.5126	−2.5491	−6.4950	−6.5121	−6.4810	−3.4755
$g_4(x)$	−0.6645	−0.6815	−0.7282	−0.6467	−0.6467	−0.7191	−0.6467	−0.6467	−0.6469	−0.6884
$f(x)$	0.0078	0.0084	0.0127	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073	0.0076

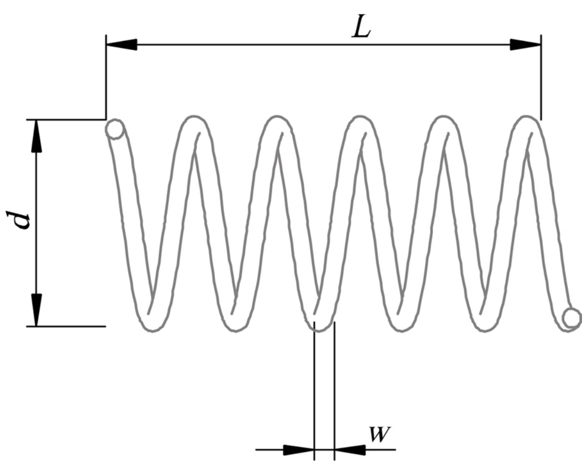

Fig. 6. Description of the tension/compression spring design problem.

Table 11
Statistical results for the tension/compression spring design problem.

Algorithm	Worst	Mean	Std	Best
ABC	0.0084	0.0076	0.0002	0.0078
DE	1.00E+06	2.00E+05	4.07E+05	0.0084
PSO	1.00E+06	1.67E+05	3.79E+05	0.0127
CS	0.0073	0.0073	2.3681e-12	0.0073
DS	0.0081	0.0074	2.2511e-04	0.0073
JADE	1.00E+06	3.333E+04	1.8257E+05	0.0073
MFO	0.0073	0.0073	1.5149e-07	0.0073
MVO	1.00E+06	4.0000e+05	4.9827e+05	0.0073
SCA	0.0077	0.0074	9.4466e-05	0.0073
EA-HC	0.0237	0.0129	0.0031	0.0076

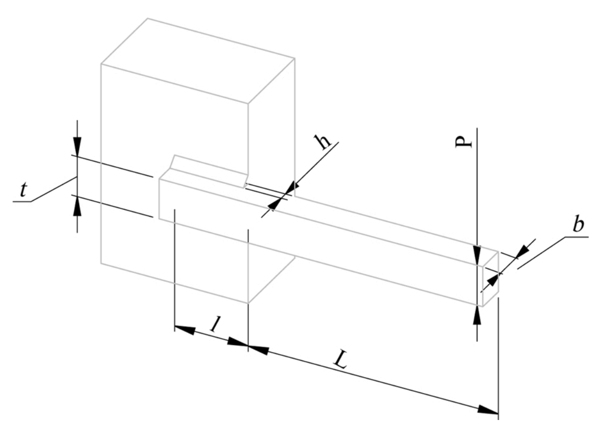

Fig. 7. Description of the Welded beam design problem.

based principles is presented. The proposed approach uses the agent-based model known as "Heroes and Cowards". Under this scheme, candidate solutions from the metaheuristic approach are considered agents while their interactions are characterized following a small set of rules that produce two emergent search processes: Coward and heroical. These procedures can be considered in terms of metaheuristic concepts as the exploration and exploitation stages. During the coward process, agents are distributed along the space as a consequence of the scape process from the enemy. Contrarily, during the hero process, agents semi-concentrate around positions marked by the agent distributions. Additionally, the model considers the use of a moderator to dynamically change between these two phases.

The performance of the proposed method is numerically compared against several state-of-art metaheuristic schemes evaluating 23 benchmark functions with different complexities. The experimental results indicate that the agent-based metaheuristic approach overcomes its competitors in terms of accuracy and scalability. The remarkable performance of the proposed method is based on the dynamic change between exploration and exploitation during the entire optimization process.

The paper has two general objectives: (I) To demonstrate the efficacy of agent-based models as metaheuristic methods; and (II) to show the promising potential in the combination of both artificial intelligence paradigms. Under the obtained results, it is clear that the use of agent-based concepts and models allows the construction of effective metaheuristic schemes. These methods maintain better innovative search patterns since they have been designed considering a well-known methodology to implement complex interactions among agents.

Authorship statement

All persons who meet authorship criteria are listed as authors, and all authors certify that they have participated sufficiently in the work to take public responsibility for the content, including participation in the concept, design, analysis, writing, or revision of the manuscript. Furthermore, each author certifies that this material or similar material has not been and will not be submitted to or published in any other publication before its appearance in the Journal of Computational Science.

Disclosure

- None of the authors of this paper has a financial or personal relationship with other people or organizations that could inappropriately influence or bias the content of the paper.
- It is to specifically state that "No Competing interests are at stake and there is No Conflict of Interest" with other people or organizations that could inappropriately influence or bias the content of the paper.

Declaration of Competing Interest

The authors report no declarations of interest.

Table 12
Numerical results for the welded beam design problem.

Parameter	ABC	DE	PSO	CS	DS	JADE	MFO	MVO	SCA	EA-HC
h	0.1993	0.1626	0.8120	0.2057	0.2057	0.2739	0.2057	0.2045	0.2004	0.2057
l	3.8115	6.8239	6.4319	3.4705	3.4706	3.4241	3.4705	3.5040	3.7893	3.4705
t	8.6442	8.7650	9.2837	9.0366	9.0385	9.0331	9.0366	9.0375	9.0748	9.0366
b	0.2259	0.2346	0.7008	0.2057	0.2057	0.2869	0.2057	0.2058	0.2062	0.2057
$g_1(x)$	−81.6604	−2977.1639	−1.16E+04	−0.0062	−0.0322	−3.3350e+03	−0.0062	−23.2984	−616.4397	−0.0062
$g_2(x)$	−135.7300	−2035.9101	−2.17E+04	−0.1334	−14.1393	−8.4778e+03	−0.1334	−9.1686	−315.5598	−0.1334
$g_3(x)$	−0.0265	−0.0720	0.1112	−1.2836e-06	−5.0225e-05	−0.0130	−1.2836e-06	−0.0012	−0.0058	−1.2836e-06
$g_4(x)$	−3.3229	−2.9372	1.4640	−3.4330	−3.4326	−2.8189	−3.4406	−3.4297	−3.3945	−3.4306
$g_5(x)$	−0.0743	−0.0376	−0.6870	−0.0807	−0.0807	−0.1489	−0.0807	−0.0795	−0.0754	−0.0807
$g_6(x)$	−0.2350	−0.2361	−0.2461	−0.2355	−0.2356	−0.2396	−0.2355	−0.2355	−0.2358	−0.2355
$g_7(x)$	−1706.7356	−2718.3738	−2.35E+05	−0.0157	−1.9242	−1.0283e+04	−0.0157	−2.5351	−55.4759	−0.0157
$f(x)$	1.8403	2.2593	2.0118	1.7249	1.7252	1.7577	1.7249	1.7278	1.7694	1.7249

Table 13
Statistical results for the welded beam design problem.

Algorithm	Worst	Mean	Std	Best
ABC	2.0848	1.9269	0.0849	1.8403
DE	3.6161	2.6481	0.3445	2.2593
PSO	3.3931	2.1544	0.3594	2.0118
CS	1.7250	1.7249	2.2364e-05	1.7249
DS	2.2552	1.8165	0.1205	1.7252
JADE	2.9267	2.2761	0.3097	1.7578
MFO	2.3171	1.7854	0.1282	1.7249
MVO	1.8169	1.7412	0.0182	1.7278
SCA	1.9711	1.8519	0.0493	1.7694
EA-HC	2.3171	1.9850	0.2046	1.7249

Appendix A. Supplementary data

Supplementary material related to this article can be found, in the online version, at doi:https://doi.org/10.1016/j.jocs.2020.101244.

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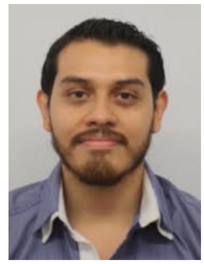

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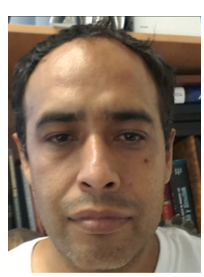

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