# Particle Swarm Optimization

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# Particle Swarm Optimization (PSO)

- Fue desarrollada por Federico Marini et. al. en el año 2015 [a].
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoreamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

[a] *Particle swarm optimization (PSO). A tutorial*, Elsevier (2015)

# PSO: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$bestX = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix} \quad (1)$$

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix} \quad (2)$$

$$vMax = [-20, 20] \quad (3)$$

- Donde:
  - *vMax* es un rango que limita la velocidad de las particulas

# PSO: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter} \quad (4)$$

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j}) \quad (5)$$

$$X_{i,j} = X_{i,j} + v_{i,j} \quad (6)$$

- Donde:
  - $wMax = 1{,}0$
  - $wMin = 0{,}2$
  - $c_1 = 2$ y $c_2 = 2$

# PSO: Pseudocódigo

**Algorithm 1** Particle Swarm Optimization

**Input:** Population $X=\{x_1, x_2, \ldots, x_N\}$
**Output:** Updated population $X'=\{x'_1, x'_2, \ldots, x'_n\}$ and Best

```
1:  Initialize random particle population X and copy in BestX; constants: vMax, wMax, wMin, c1 and c2
2:  for it = 1 to  MaxIt do
3:          calulate the fitness of each X
4:          select Best
5:          Check for each element in BestX if X[i] is better, then update BestX[i] accordingly
6:          v = zeros(N,dim)
7:          w = wMax - it * ((wMax - wMin) / MaxIter)
8:          for i=0 to N do
9:                  for j=0 to dim do
10:                         r1,r2 = random()
11:                         v[i,j] = w*v[i,j] + c1*r1*(BestX[i,j]-X[i,j]) + c2*r2*(Best[j]-X[i,j])
12:                         Ensure v stays within the range [-vMax, vMax]; if exceeded, adjust v to the nearest limit
13:                         X[i,j] = X[i,j] + v[i,j]
14: return updated population X' and Best
```

# PSO: Ejemplo prático - parámetros iniciales

Considerando

$$\text{Min } z = x_1^2 + x_2^2$$

Sujeto a

$$x_1, x_2 \in [-100, 100]$$

Configuración inicial de PSO:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 5000 iteraciones
- $vMax = 20$
- $wMax = 1{,}0$
- $wMin = 0{,}2$
- $c_1 = 2$
- $c_2 = 2$

# PSO: Ejemplo prático - soluciones inciales

Soluciones inciales:
ind 1: [ 65.5597, -89.2108] / fitness: 12256.6410
ind 2: [-68.0593, -9.6881] / fitness: 4725.9244
ind 3: [36.1261, 63.6346] / fitness: 5354.4598
ind 4: [-89.2303, 59.4155] / fitness: 11492.2387

Mejor solución:
ind 2: [-68.0593, -9.6881] / fitness: 4725.9244

# PSO: Ejemplo práctico - iter 1

Ecuaciones generales de la iteración 1:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$

$$bestX = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix}$$

$$w = 1{,}0 - 1 \cdot \frac{1{,}0 - 0{,}2}{5000} = 0{,}9998$$

$$bestX = \begin{bmatrix} 65{,}5597 & -89{,}2108 \\ -68{,}0593 & -9{,}6881 \\ 36{,}1261 & 63{,}6346 \\ -89{,}2303 & 59{,}4155 \end{bmatrix}$$

# PSO: Ejemplo prático - ind 1 - dim 1 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}4119$
$r_2 = random(0,1) = 0{,}1678$
$v_{1,1} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}4119 \cdot (65{,}5597 - 65{,}5597) + 2 \cdot 0{,}1678 \cdot (-68{,}0593 - 65{,}5597) = -44{,}8293$

$v_{1,1} < -vMax$
$-44{,}8293 < -20$
$v_{1,1} = -vMax = -20$
$X_{1,1} = 65{,}5597 - 20{,}0000 = 45{,}5597$

# PSO: Ejemplo prático - ind 1 - dim 2 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}1958$
$r_2 = random(0,1) = 0{,}2073$
$v_{1,2} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}1958 \cdot (-89{,}2108 + 89{,}2108) + 2 \cdot 0{,}2073 \cdot (-9{,}6881 + 89{,}2108) = 32{,}9715$

$v_{1,2} > vMax$
$32{,}9715 > 20$
$v_{1,2} = vMax = 20$
$X_{1,2} = -89{,}2108 + 20{,}0000 = -69{,}2108$

# PSO: Ejemplo prático - ind - 2 dim 1 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}6211$
$r_2 = random(0,1) = 0{,}2664$
$v_{2,1} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}6211 \cdot (-68{,}0593 + 68{,}0593) + 2 \cdot 0{,}2664 \cdot (-68{,}0593 + 68{,}0593) = 0{,}0000$

$X_{2,1} = -68{,}0593 + 0{,}0000 = -68{,}0593$

# PSO: Ejemplo prático - ind 2 - dim 2 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}1452$
$r_2 = random(0,1) = 0{,}4729$
$v_{2,2} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}1452 \cdot (-9{,}6881 + 9{,}6881) + 2 \cdot 0{,}4729 \cdot (-9{,}6881 + 9{,}6881) = 0{,}0000$

$X_{2,2} = -9{,}6881 + 0{,}0000 = -9{,}6881$

# PSO: Ejemplo prático - ind 3 - dim 1 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}4425$
$r_2 = random(0,1) = 0{,}9511$
$v_{3,1} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}4425 \cdot (36{,}1261 - 36{,}1261) + 2 \cdot 0{,}9511 \cdot (-68{,}0593 - 36{,}1261) = -198{,}1876$

$v_{3,1} < -vMax$
$-198{,}1876 < -20$
$v_{3,1} = vMax = -20$
$X_{3,1} = 36{,}1261 + -20{,}0000 = 16{,}1261$

# PSO: Ejemplo prático - ind 3 - dim 2 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}7923$
$r_2 = random(0,1) = 0{,}2359$
$v_{1,1} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}7923 \cdot (63{,}6346 - 63{,}6346) + 2 \cdot 0{,}2359 \cdot (-9{,}6881 - 63{,}6346) = -34{,}5917$

$v_{3,2} < -vMax$
$-34{,}5917 < -20$
$v_{3,2} = vMax = -20$
$X_{3,2} = 63{,}6346 + -20{,}0000 = 43{,}6346$

# PSO: Ejemplo prático - ind 4 - dim 1 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}7301$
$r_2 = random(0,1) = 0{,}1236$
$v_{4,1} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}7301 \cdot (-89{,}2303 + 89{,}2303) + 2 \cdot 0{,}1236 \cdot (-68{,}0593 + 89{,}2303) = 5{,}2318$

$X_{4,1} = -89{,}2303 + 5{,}2318 = -83{,}9985$

# PSO: Ejemplo prático - ind 4 - dim 2 - iter 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}2531$
$r_2 = random(0,1) = 0{,}3419$
$v_{4,2} = 0{,}9998 \cdot 0{,}0000 + 2 \cdot 0{,}2531 \cdot (59{,}4155 - 59{,}4155) + 2 \cdot 0{,}3419 \cdot (-9{,}6881 - 59{,}4155) = -47{,}2525$

$v_{4,2} < -vMax$
$-47{,}2525 < -20$
$v_{4,2} = vMax = -20$
$X_{4,2} = 59{,}4155 + -20{,}0000 = 39{,}4155$

# PSO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 1:
ind 1: [ 45.5597, -69.2108], infactibles: 0
ind 2: [-68.0593, -9.6881], infactibles: 0
ind 3: [16.1261, 43.6346], infactibles: 0
ind 4: [-83.9985, 39.4155], infactibles: 0

Reparación de soluciones:
ind 1: [ 45.5597, -69.2108] / fitness: 6865.8210
ind 2: [-68.0593, -9.6881] / fitness: 4725.9244
ind 3: [16.1261, 43.6346] / fitness: 2164.0316
ind 4: [-83.9985, 39.4155] / fitness: 8609.3302

Mejor solución:
ind 3: [16.1261, 43.6346] / fitness: 2164.0316

# PSO: Ejemplo práctico - iter 2

Ecuaciones generales de la iteración 2:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$

$$bestX = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix}$$

$$w = 1{,}0 - 2 \cdot \frac{1{,}0 - 0{,}2}{5000} = 0{,}9997$$

$$bestX = \begin{bmatrix} 45{,}5597 & -69{,}2108 \\ -68{,}0593 & -9{,}6881 \\ 16{,}1261 & 43{,}6346 \\ -83{,}9985 & 39{,}4155 \end{bmatrix}$$

# PSO: Ejemplo prático - ind 1 - dim 1 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}1398$
$r_2 = random(0,1) = 0{,}1348$
$v_{1,1} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}1398 \cdot (45{,}5597 - 45{,}5597) + 2 \cdot 0{,}1348 \cdot (16{,}1261 - 45{,}5597) = -7{,}9363$

$X_{1,1} = 45{,}5597 + -7{,}9363 = 37{,}6234$

# PSO: Ejemplo prático - ind 1 - dim 2 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}7543$
$r_2 = random(0,1) = 0{,}7844$
$v_{1,2} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}7543 \cdot (-69{,}2108 + 69{,}2108) + 2 \cdot 0{,}7844 \cdot (43{,}6346 + 69{,}2108) = 177{,}0244$

$v_{1,2} > vMax$
$177{,}0244 > 20$
$v_{1,2} = vMax = 20$
$X_{1,2} = -69{,}2108 + 20{,}0000 = -49{,}2108$

# PSO: Ejemplo prático - ind 2 - dim 1 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}3527$
$r_2 = random(0,1) = 0{,}9777$
$v_{2,1} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}3527 \cdot (-68{,}0593 + 68{,}0593) + 2 \cdot 0{,}9777 \cdot (16{,}1261 + 68{,}0593) = 164{,}6212$

$v_{2,1} > vMax$
$164{,}6212 > 20$
$v_{2,1} = vMax = 20$
$X_{2,1} = -68{,}0593 + 20{,}0000 = -48{,}0593$

# PSO: Ejemplo prático - ind 2 - dim 2 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}6600$
$r_2 = random(0,1) = 0{,}3774$
$v_{2,2} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}6600 \cdot (-9{,}6881 + 9{,}6881) + 2 \cdot 0{,}3774 \cdot (43{,}6346 + 9{,}6881) = 40{,}2440$

$v_{2,2} > vMax$
$40{,}2440 > 20$
$v_{2,2} = vMax = 20$
$X_{2,2} = -9{,}6881 + 20{,}0000 = 10{,}3119$

# PSO: Ejemplo prático - ind 3 - dim 1 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}3652$
$r_2 = random(0,1) = 0{,}0702$
$v_{3,1} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}3652 \cdot (16{,}1261 - 16{,}1261) + 2 \cdot 0{,}0702 \cdot (16{,}1261 - 16{,}1261) = 0{,}0000$

$X_{3,1} = 16{,}1261 + 0{,}0000 = 16{,}1261$

# PSO: Ejemplo prático - ind 3 - dim 2 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}4273$
$r_2 = random(0,1) = 0{,}8131$
$v_{3,2} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}4273 \cdot (43{,}6346 - 43{,}6346) + 2 \cdot 0{,}8131 \cdot (43{,}6346 - 43{,}6346) = 0{,}0000$

$X_{3,2} = 43{,}6346 + 0{,}0000 = 43{,}6346$

# PSO: Ejemplo prático - ind 4 - dim 1 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}0994$
$r_2 = random(0,1) = 0{,}2623$
$v_{4,1} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}0994 \cdot (-83{,}9985 + 83{,}9985) + 2 \cdot 0{,}2623 \cdot (16{,}1261 + 83{,}9985) = 52{,}5290$

$v_{4,1} > vMax$
$52{,}5290 > 20$
$v_{4,1} = vMax = 20$
$X_{4,1} = -83{,}9985 + 20{,}0000 = -63{,}9985$

# PSO: Ejemplo prático - ind 4 - dim 2 - iter 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}4001$
$r_2 = random(0,1) = 0{,}5388$
$v_{4,2} = 0{,}9997 \cdot 0{,}0000 + 2 \cdot 0{,}4001 \cdot (39{,}4155 - 39{,}4155) + 2 \cdot 0{,}5388 \cdot (43{,}6346 - 39{,}4155) = 4{,}5466$

$X_{4,2} = 39{,}4155 + 4{,}5466 = 43{,}9620$

# PSO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 2:
ind 1: [ 37.6234, -49.2108], infactibles: 0
ind 2: [-48.0593, 10.3119], infactibles: 0
ind 3: [16.1261, 43.6346], infactibles: 0
ind 4: [-63.9985, 43.962 ], infactibles: 0

Reparación de soluciones:
ind 1: [ 37.6234, -49.2108] / fitness: 3837.2256
ind 2: [-48.0593, 10.3119] / fitness: 2416.0277
ind 3: [16.1261, 43.6346] / fitness: 2164.0316
ind 4: [-63.9985, 43.962 ] / fitness: 6028.4711

Mejor solución:
ind 3: [16.1261, 43.6346] / fitness: 2164.0316

# PSO: Ejemplo práctico - iter 5000

Ecuaciones generales de la iteración 5000:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$

$$bestX = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix}$$

$$w = 1{,}0 - 5000 \cdot \frac{1{,}0 - 0{,}2}{5000} = 0{,}2000$$

$$bestX = \begin{bmatrix} -0{,}3212 & 0{,}0208 \\ -0{,}3212 & 0{,}0208 \\ -0{,}3212 & 0{,}0208 \\ -0{,}3212 & 0{,}0208 \end{bmatrix}$$

# PSO: Ejemplo prático - ind - 1 dim 1 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}8887$
$r_2 = random(0,1) = 0{,}5080$
$v_{1,1} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}8887 \cdot (-0{,}3212 + 0{,}3212) + 2 \cdot 0{,}5080 \cdot (-0{,}3212 + 0{,}3212) = 0{,}0000$

$X_{1,1} = -0{,}3212 + 0{,}0000 = -0{,}3212$

# PSO: Ejemplo prático - ind 1 - dim 2 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}1081$
$r_2 = random(0,1) = 0{,}2942$
$v_{1,2} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}1081 \cdot (0{,}0208 - 0{,}0208) + 2 \cdot 0{,}2942 \cdot (0{,}0208 - 0{,}0208) = 0{,}0000$

$X_{1,2} = 0{,}0208 + 0{,}0000 = 0{,}0208$

# PSO: Ejemplo prático - ind 2 - dim 1 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}1395$
$r_2 = random(0,1) = 0{,}1656$
$v_{2,1} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}1395 \cdot (-0{,}3212 + 0{,}3212) + 2 \cdot 0{,}1656 \cdot (-0{,}3212 + 0{,}3212) = 0{,}0000$

$X_{2,1} = -0{,}3212 + 0{,}0000 = -0{,}3212$

# PSO: Ejemplo prático - ind 2 - dim 2 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}6422$
$r_2 = random(0,1) = 0{,}0291$
$v_{2,2} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}6422 \cdot (0{,}0208 - 0{,}0208) + 2 \cdot 0{,}0291 \cdot (0{,}0208 - 0{,}0208) = 0{,}0000$

$X_{2,2} = 0{,}0208 + 0{,}0000 = 0{,}0208$

# PSO: Ejemplo prático - ind 3 - dim 1 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}4158$
$r_2 = random(0,1) = 0{,}3688$
$v_{3,1} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}4158 \cdot (-0{,}3212 + 0{,}3212) + 2 \cdot 0{,}3688 \cdot (-0{,}3212 + 0{,}3212) = 0{,}0000$

$X_{3,1} = -0{,}3212 + 0{,}0000 = -0{,}3212$

# PSO: Ejemplo prático - ind 3 - dim 2 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}7118$
$r_2 = random(0,1) = 0{,}2098$
$v_{3,2} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}7118 \cdot (0{,}0208 - 0{,}0208) + 2 \cdot 0{,}2098 \cdot (0{,}0208 - 0{,}0208) = 0{,}0000$

$X_{3,2} = 0{,}0208 + 0{,}0000 = 0{,}0208$

# PSO: Ejemplo prático - ind 4 - dim 1 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}7971$
$r_2 = random(0,1) = 0{,}4384$
$v_{4,1} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}7971 \cdot (-0{,}3212 + 0{,}3212) + 2 \cdot 0{,}4384 \cdot (-0{,}3212 + 0{,}3212) = 0{,}0000$

$X_{4,1} = -0{,}3212 + 0{,}0000 = -0{,}3212$

# PSO: Ejemplo prático - ind 4 - dim 2 - iter 5000

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$r_1 = random(0,1) = 0{,}5305$
$r_2 = random(0,1) = 0{,}0261$
$v_{4,2} = 0{,}2000 \cdot 0{,}0000 + 2 \cdot 0{,}5305 \cdot (0{,}0208 - 0{,}0208) + 2 \cdot 0{,}0261 \cdot (0{,}0208 - 0{,}0208) = 0{,}0000$

$X_{4,2} = 0{,}0208 + 0{,}0000 = 0{,}0208$

# PSO: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soliciones obtenidas en la iteración 5000:
ind 1: [-0.3212, 0.0208], infactibles: 0
ind 2: [-0.3212, 0.0208], infactibles: 0
ind 3: [-0.3212, 0.0208], infactibles: 0
ind 4: [-0.3212, 0.0208], infactibles: 0

Reparación de soluciones:
ind 1: [-0.3212, 0.0208] / fitness: 0.1036
ind 2: [-0.3212, 0.0208] / fitness: 0.1036
ind 3: [-0.3212, 0.0208] / fitness: 0.1036
ind 4: [-0.3212, 0.0208] / fitness: 0.1036

Mejor solución:
ind 1: [-0.3212, 0.0208] / fitness: 0.1036