

Pendulum Search Algorithm

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Pendulum Search Algorithm (PSA)

- Fue desarrollada por Nor Azlina Ab. Aziz et. al. en el año 2022 ^a.
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoriamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

^a*Pendulum Search Algorithm: An Optimization Algorithm Based on Simple Harmonic Motion and Its Application for a Vaccine Distribution Problem*, Algorithms (2022)

- Ecuaciones de movimientos general

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t) \quad (1)$$

$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand)) \quad (2)$$

- Donde:

- $X_{i,j}^t$ es la posición de la i-ésima solución en la j-ésima dimensión y en la t-ésima iteración
- $Best_j^t$ es la mejor solución en la j-ésima dimensión y en la t-ésima iteración
- $pend_{i,j}^t$, es un parámetro donde:
 - t es la iteración actual
 - t_{max} es el número total de iteraciones
 - $rand$ es un número aleatorio entre $[0, 1]$

Algorithm 1 Pendulum Search Algorithm

Input: The population $X = \{X_1, X_2, \dots, X_m\}$

Output: The updated population $X' = \{X'_1, X'_2, \dots, X'_m\}$ and $Best^t$

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1: Initialize random population X
2: Evaluate the solutions in the objective function
3: Update  $Best^t$ 
4: for  $t = 1$  to  $T$  do
5:   for  $i = 1$  to  $m$  do
6:     for  $j = 1$  to  $D$  do                                ▷  $D = \text{number of dimensions}$ 
7:        $pend_{i,j}^t = 2 \cdot \exp(\frac{-t}{Tmax}) \cdot (\cos(2 \cdot \pi \cdot rand))$ 
8:        $X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$ 
9:     end for
10:   end for
11:   Evaluate the solutions in the objective function
12:   Update  $Best^t$ 
13: end for
14: Return the updated population  $X'$  where  $Best$  is the best result
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PSA: Ejemplo práctico - parámetros iniciales

Considerando

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2$$

Sujeto a

$$x_1, x_2, x_3 \in [-100, 100]$$

Configuración inicial de PSA:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 100 iteraciones
- Constante $a = 2$

PSA: Ejemplo práctico - soluciones iniciales

Soluciones iniciales:

ind 1: [-33.7939, 94.2896, 38.8342] / fitness: 11540.6514

ind 2: [53.5519, 7.0044, -91.4171] / fitness: 11273.9538

ind 3: [-93.6863, -78.8417, -19.722] / fitness: 15382.0938

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

PSA: Ejemplo práctico - iter 1 - ind 1 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,1}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,3094))$$

$$pend_{1,1}^1 = -0,7292 / X_{1,1}^1 = -33,7939 / Best_1^1 = 24,8353$$

$$X_{1,1}^2 = X_{1,1}^1 + pend_{1,1}^1 \cdot (Best_1^1 - X_{1,1}^1)$$

$$X_{1,1}^2 = -33,7939 - 0,7292 \cdot (24,8353 + 33,7939)$$

$$X_{1,1}^2 = -76,5463$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,2}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,0177))$$

$$pend_{1,2}^1 = 1,9876 / X_{1,2}^1 = 94,2896 / Best_2^1 = -38,6385$$

$$X_{1,2}^2 = X_{1,2}^1 + pend_{1,2}^1 \cdot (Best_2^1 - X_{1,2}^1)$$

$$X_{1,2}^2 = 94,2896 + 1,9876 \cdot (-38,6385 - 94,2896)$$

$$X_{1,2}^2 = -169,9183$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,3}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,0553))$$

$$pend_{1,3}^1 = 1,8805 / X_{1,3}^1 = 38,8342 / Best_3^1 = 77,9742$$

$$X_{1,3}^2 = X_{1,3}^1 + pend_{1,3}^1 \cdot (Best_3^1 - X_{1,3}^1)$$

$$X_{1,3}^2 = 38,8342 + 1,8805 \cdot (77,9742 - 38,8342)$$

$$X_{1,3}^2 = 112,437$$

PSA: Ejemplo práctico - iter 1 - ind 2 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,1}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,8257))$$

$$pend_{2,1}^1 = 0,9158 / X_{2,1}^1 = 53,5519 / Best_1^1 = 24,8353$$

$$X_{2,1}^2 = X_{2,1}^1 + pend_{2,1}^1 \cdot (Best_1^1 - X_{2,1}^1)$$

$$X_{2,1}^2 = 53,5519 + 0,9158 \cdot (24,8353 - 53,5519)$$

$$X_{2,1}^2 = 27,2532$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,2}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,7772))$$

$$pend_{2,2}^1 = 0,3401 / X_{2,2}^1 = 7,0044 / Best_2^1 = -38,6385$$

$$X_{2,2}^2 = X_{2,2}^1 + pend_{2,2}^1 \cdot (Best_2^1 - X_{2,2}^1)$$

$$X_{2,2}^2 = 7,0044 + 0,3401 \cdot (-38,6385 - 7,0044)$$

$$X_{2,2}^2 = -8,5188$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,3}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,4459))$$

$$pend_{2,3}^1 = -1,8856 / X_{2,3}^1 = -91,4171 / Best_3^1 = 77,9742$$

$$X_{2,3}^2 = X_{2,3}^1 + pend_{2,3}^1 \cdot (Best_3^1 - X_{2,3}^1)$$

$$X_{2,3}^2 = -91,4171 - 1,8856 \cdot (77,9742 + 91,4171)$$

$$X_{2,3}^2 = -410,8213$$

PSA: Ejemplo práctico - iter 1 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,6315))$$

$$pend_{3,1}^1 = -1,3553 / X_{3,1}^1 = -93,6863 / Best_1^1 = 24,8353$$

$$X_{3,1}^2 = X_{3,1}^1 + pend_{3,1}^1 \cdot (Best_1^1 - X_{3,1}^1)$$

$$X_{3,1}^2 = -93,6863 - 1,3553 \cdot (24,8353 + 93,6863)$$

$$X_{3,1}^2 = -254,3186$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,2}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,5219))$$

$$pend_{3,2}^1 = -1,9811 / X_{3,2}^1 = -78,8417 / Best_2^1 = -38,6385$$

$$X_{3,2}^2 = X_{3,2}^1 + pend_{3,2}^1 \cdot (Best_2^1 - X_{3,2}^1)$$

$$X_{3,2}^2 = -78,8417 - 1,9811 \cdot (-38,6385 + 78,8417)$$

$$X_{3,2}^2 = -158,4883$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,3}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,2278))$$

$$pend_{3,3}^1 = 0,2781 / X_{3,3}^1 = -19,722 / Best_3^1 = 77,9742$$

$$X_{3,3}^2 = X_{3,3}^1 + pend_{3,3}^1 \cdot (Best_3^1 - X_{3,3}^1)$$

$$X_{3,3}^2 = -19,722 + 0,2781 \cdot (77,9742 + 19,722)$$

$$X_{3,3}^2 = 7,4473$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,1}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,4123))$$

$$pend_{4,1}^1 = -1,704 / X_{4,1}^1 = 24,8353 / Best_1^1 = 24,8353$$

$$X_{4,1}^2 = X_{4,1}^1 + pend_{4,1}^1 \cdot (Best_1^1 - X_{4,1}^1)$$

$$X_{4,1}^2 = 24,8353 - 1,704 \cdot (24,8353 - 24,8353)$$

$$X_{4,1}^2 = 24,8353$$

PSA: Ejemplo práctico - iter 1 - ind 4 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,2}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,5089))$$

$$pend_{4,2}^1 = -1,9969 / X_{4,2}^1 = -38,6385 / Best_2^1 = -38,6385$$

$$X_{4,2}^2 = X_{4,2}^1 + pend_{4,2}^1 \cdot (Best_2^1 - X_{4,2}^1)$$

$$X_{4,2}^2 = -38,6385 - 1,9969 \cdot (-38,6385 + 38,6385)$$

$$X_{4,2}^2 = -38,6385$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^1 = 2 \cdot \exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0,9267))$$

$$pend_{4,3}^1 = 1,7916 / X_{4,3}^1 = 77,9742 / Best_3^1 = 77,9742$$

$$X_{4,3}^2 = X_{4,3}^1 + pend_{4,3}^1 \cdot (Best_3^1 - X_{4,3}^1)$$

$$X_{4,3}^2 = 77,9742 + 1,7916 \cdot (77,9742 - 77,9742)$$

$$X_{4,3}^2 = 77,9742$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:

ind 1: [-76.5463, **-169.9183**, **112.437**], **dos dimensiones infactibles**

ind 2: [27.2532, -8.5188, **-410.8213**], **una dimensión infactible**

ind 3: [**-254.3186**, **-158.4883**, 7.4473], **dos dimensiones infactibles**

ind 4: [24.8353, -38.6385, 77.9742], **solución factible**

Reparación de soluciones:

ind 1: [-76.5463, -100.0, 100.0] / fitness: 25859.336

ind 2: [27.2532, -8.5188, -100.0] / fitness: 10815.3069

ind 3: [-100.0, -100.0, 7.4473] / fitness: 20055.4623

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,1}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,7986))$$

$$pend_{1,1}^2 = 0,5953 / X_{1,1}^2 = -76,5463 / Best_1^2 = 24,8353$$

$$X_{1,1}^3 = X_{1,1}^2 + pend_{1,1}^2 \cdot (Best_1^2 - X_{1,1}^2)$$

$$X_{1,1}^3 = -76,5463 + 0,5953 \cdot (24,8353 + 76,5463)$$

$$X_{1,1}^3 = -16,1938$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,2}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,4083))$$

$$pend_{1,2}^2 = -1,6604 / X_{1,2}^2 = -100,0 / Best_2^2 = -38,6385$$

$$X_{1,2}^3 = X_{1,2}^2 + pend_{1,2}^2 \cdot (Best_2^2 - X_{1,2}^2)$$

$$X_{1,2}^3 = -100,0 - 1,6604 \cdot (-38,6385 + 100,0)$$

$$X_{1,2}^3 = -201,8846$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,3}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,867))$$
$$pend_{1,3}^2 = 1,328 / X_{1,3}^2 = 100,0 / Best_3^2 = 77,9742$$

$$X_{1,3}^3 = X_{1,3}^2 + pend_{1,3}^2 \cdot (Best_3^2 - X_{1,3}^2)$$
$$X_{1,3}^3 = 100,0 + 1,328 \cdot (77,9742 - 100,0)$$
$$X_{1,3}^3 = 70,7497$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,1}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,7687))$$

$$pend_{2,1}^2 = 0,2321 / X_{2,1}^2 = 27,2532 / Best_1^2 = 24,8353$$

$$X_{2,1}^3 = X_{2,1}^2 + pend_{2,1}^2 \cdot (Best_1^2 - X_{2,1}^2)$$

$$X_{2,1}^3 = 27,2532 + 0,2321 \cdot (24,8353 - 27,2532)$$

$$X_{2,1}^3 = 26,692$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,2}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,323))$$
$$pend_{2,2}^2 = -0,8767 / X_{2,2}^2 = -8,5188 / Best_2^2 = -38,6385$$

$$X_{2,2}^3 = X_{2,2}^2 + pend_{2,2}^2 \cdot (Best_2^2 - X_{2,2}^2)$$
$$X_{2,2}^3 = -8,5188 - 0,8767 \cdot (-38,6385 + 8,5188)$$
$$X_{2,2}^3 = 17,8871$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,3}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,5507))$$
$$pend_{2,3}^2 = -1,8805 / X_{2,3}^2 = -100,0 / Best_3^2 = 77,9742$$

$$X_{2,3}^3 = X_{2,3}^2 + pend_{2,3}^2 \cdot (Best_3^2 - X_{2,3}^2)$$
$$X_{2,3}^3 = -100,0 - 1,8805 \cdot (77,9742 + 100,0)$$
$$X_{2,3}^3 = -434,6805$$

PSA: Ejemplo práctico - iter 2 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,1147))$$

$$pend_{3,1}^2 = 1,4878 / X_{3,1}^2 = -100,0 / Best_1^2 = 24,8353$$

$$X_{3,1}^3 = X_{3,1}^2 + pend_{3,1}^2 \cdot (Best_1^2 - X_{3,1}^2)$$

$$X_{3,1}^3 = -100,0 + 1,4878 \cdot (24,8353 + 100,0)$$

$$X_{3,1}^3 = 85,73$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,2}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,7433))$$

$$pend_{3,2}^2 = -0,0833 / X_{3,2}^2 = -100,0 / Best_2^2 = -38,6385$$

$$X_{3,2}^3 = X_{3,2}^2 + pend_{3,2}^2 \cdot (Best_2^2 - X_{3,2}^2)$$

$$X_{3,2}^3 = -100,0 - 0,0833 \cdot (-38,6385 + 100,0)$$

$$X_{3,2}^3 = -105,1114$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,3}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,6752))$$

$$pend_{3,3}^2 = -0,8967 / X_{3,3}^2 = 7,4473 / Best_3^2 = 77,9742$$

$$X_{3,3}^3 = X_{3,3}^2 + pend_{3,3}^2 \cdot (Best_3^2 - X_{3,3}^2)$$

$$X_{3,3}^3 = 7,4473 - 0,8967 \cdot (77,9742 - 7,4473)$$

$$X_{3,3}^3 = -55,7942$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,1}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,2914))$$

$$pend_{4,1}^2 = -0,5093 / X_{4,1}^2 = 24,8353 / Best_1^2 = 24,8353$$

$$X_{4,1}^3 = X_{4,1}^2 + pend_{4,1}^2 \cdot (Best_1^2 - X_{4,1}^2)$$

$$X_{4,1}^3 = 24,8353 - 0,5093 \cdot (24,8353 - 24,8353)$$

$$X_{4,1}^3 = 24,8353$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,2}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,3383))$$

$$pend_{4,2}^2 = -1,0431 / X_{4,2}^2 = -38,6385 / Best_2^2 = -38,6385$$

$$X_{4,2}^3 = X_{4,2}^2 + pend_{4,2}^2 \cdot (Best_2^2 - X_{4,2}^2)$$

$$X_{4,2}^3 = -38,6385 - 1,0431 \cdot (-38,6385 + 38,6385)$$

$$X_{4,2}^3 = -38,6385$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^2 = 2 \cdot \exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0,1651))$$

$$pend_{4,3}^2 = 1,0069 / X_{4,3}^2 = 77,9742 / Best_3^2 = 77,9742$$

$$X_{4,3}^3 = X_{4,3}^2 + pend_{4,3}^2 \cdot (Best_3^2 - X_{4,3}^2)$$

$$X_{4,3}^3 = 77,9742 + 1,0069 \cdot (77,9742 - 77,9742)$$

$$X_{4,3}^3 = 77,9742$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:

ind 1: [-16.1938, **-201.8846**, 70.7497], **una dimensión factible**

ind 2: [26.692, 17.8871, **-434.6805**], **una dimensión factible**

ind 3: [85.73, **-105.1114**, -55.7942], **una dimensión factible**

ind 4: [24.8353, -38.6385, 77.9742], **solución factible**

Reparación de soluciones:

ind 1: [-16.1938, -100.0, 70.7497] / fitness: 15267.7592

ind 2: [26.692, 17.8871, -100.0] / fitness: 11032.4112

ind 3: [85.73, -100.0, -55.7942] / fitness: 20462.6257

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:

ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

PSA: Ejemplo práctico - iter 100 - ind 1 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,1}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,6355))$$

$$pend_{1,1}^{100} = -0,4897 / X_{1,1}^{100} = 0,0515 / Best_1^{100} = 0,0515$$

$$X_{1,1}^{101} = X_{1,1}^{100} + pend_{1,1}^{100} \cdot (Best_1^{100} - X_{1,1}^{100})$$

$$X_{1,1}^{101} = 0,0515 - 0,4897 \cdot (0,0515 - 0,0515)$$

$$X_{1,1}^{101} = 0,0515$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,2}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,4321))$$
$$pend_{1,2}^{100} = -0,6765 / X_{1,2}^{100} = -0,074 / Best_2^{100} = -0,074$$

$$X_{1,2}^{101} = X_{1,2}^{100} + pend_{1,2}^{100} \cdot (Best_2^{100} - X_{1,2}^{100})$$
$$X_{1,2}^{101} = -0,074 - 0,6765 \cdot (-0,074 + 0,074)$$
$$X_{1,2}^{101} = -0,074$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,3}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,5042))$$
$$pend_{1,3}^{100} = -0,7429 / X_{1,3}^{100} = 0,9181 / Best_3^{100} = 0,9181$$

$$X_{1,3}^{101} = X_{1,3}^{100} + pend_{1,3}^{100} \cdot (Best_3^{100} - X_{1,3}^{100})$$
$$X_{1,3}^{101} = 0,9181 - 0,7429 \cdot (0,9181 - 0,9181)$$
$$X_{1,3}^{101} = 0,9181$$

PSA: Ejemplo práctico - iter 100 - ind 2 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,1}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,3462))$$

$$pend_{2,1}^{100} = -0,4223 / X_{2,1}^{100} = 0,0515 / Best_1^{100} = 0,0515$$

$$X_{2,1}^{101} = X_{2,1}^{100} + pend_{2,1}^{100} \cdot (Best_1^{100} - X_{2,1}^{100})$$

$$X_{2,1}^{101} = 0,0515 - 0,4223 \cdot (0,0515 - 0,0515)$$

$$X_{2,1}^{101} = 0,0515$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,2}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,3179))$$
$$pend_{2,2}^{100} = -0,3075 / X_{2,2}^{100} = -0,074 / Best_2^{100} = -0,074$$

$$X_{2,2}^{101} = X_{2,2}^{100} + pend_{2,2}^{100} \cdot (Best_2^{100} - X_{2,2}^{100})$$
$$X_{2,2}^{101} = -0,074 - 0,3075 \cdot (-0,074 + 0,074)$$
$$X_{2,2}^{101} = -0,074$$

PSA: Ejemplo práctico - iter 100 - ind 2 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,3}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,0765))$$
$$pend_{2,3}^{100} = 0,6589 / X_{2,3}^{100} = 0,9181 / Best_3^{100} = 0,9181$$

$$X_{2,3}^{101} = X_{2,3}^{100} + pend_{2,3}^{100} \cdot (Best_3^{100} - X_{2,3}^{100})$$
$$X_{2,3}^{101} = 0,9181 + 0,6589 \cdot (0,9181 - 0,9181)$$
$$X_{2,3}^{101} = 0,9181$$

PSA: Ejemplo práctico - iter 100 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,4507))$$

$$pend_{3,1}^{100} = -0,7078 / X_{3,1}^{100} = 0,0515 / Best_1^{100} = 0,0515$$

$$X_{3,1}^{101} = X_{3,1}^{100} + pend_{3,1}^{100} \cdot (Best_1^{100} - X_{3,1}^{100})$$

$$X_{3,1}^{101} = 0,0515 - 0,7078 \cdot (0,0515 - 0,0515)$$

$$X_{3,1}^{101} = 0,0515$$

PSA: Ejemplo práctico - iter 100 - ind 3 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,2}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,741))$$

$$pend_{3,2}^{100} = -0,042 / X_{3,2}^{100} = -0,074 / Best_2^{100} = -0,074$$

$$X_{3,2}^{101} = X_{3,2}^{100} + pend_{3,2}^{100} \cdot (Best_2^{100} - X_{3,2}^{100})$$

$$X_{3,2}^{101} = -0,074 - 0,042 \cdot (-0,074 + 0,074)$$

$$X_{3,2}^{101} = -0,074$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,3}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,5312))$$
$$pend_{3,3}^{100} = -0,7289 / X_{3,3}^{100} = 0,9181 / Best_3^{100} = 0,9181$$

$$X_{3,3}^{101} = X_{3,3}^{100} + pend_{3,3}^{100} \cdot (Best_3^{100} - X_{3,3}^{100})$$
$$X_{3,3}^{101} = 0,9181 - 0,7289 \cdot (0,9181 - 0,9181)$$
$$X_{3,3}^{101} = 0,9181$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,1}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,8716))$$
$$pend_{4,1}^{100} = 0,5141 / X_{4,1}^{100} = 0,0515 / Best_1^{100} = 0,0515$$

$$X_{4,1}^{101} = X_{4,1}^{100} + pend_{4,1}^{100} \cdot (Best_1^{100} - X_{4,1}^{100})$$
$$X_{4,1}^{101} = 0,0515 + 0,5141 \cdot (0,0515 - 0,0515)$$
$$X_{4,1}^{101} = 0,0515$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,2}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,6165))$$

$$pend_{4,2}^{100} = -0,5528 / X_{4,2}^{100} = -0,0752 / Best_2^{100} = -0,074$$

$$X_{4,2}^{101} = X_{4,2}^{100} + pend_{4,2}^{100} \cdot (Best_2^{100} - X_{4,2}^{100})$$

$$X_{4,2}^{101} = -0,0752 - 0,5528 \cdot (-0,074 + 0,0752)$$

$$X_{4,2}^{101} = -0,0759$$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot (Best_j^t - X_{i,j}^t)$$
$$pend_{i,j}^t = 2 \cdot \exp\left(\frac{-t}{t_{max}}\right) \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^{100} = 2 \cdot \exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0,3904))$$
$$pend_{4,3}^{100} = -0,5738 / X_{4,3}^{100} = 0,9181 / Best_3^{100} = 0,9181$$

$$X_{4,3}^{101} = X_{4,3}^{100} + pend_{4,3}^{100} \cdot (Best_3^{100} - X_{4,3}^{100})$$
$$X_{4,3}^{101} = 0,9181 - 0,5738 \cdot (0,9181 - 0,9181)$$
$$X_{4,3}^{101} = 0,9181$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 2:

- ind 1: [0.0515, -0.074, 0.9181], solución factible
- ind 2: [0.0515, -0.074, 0.9181], solución factible
- ind 3: [0.0515, -0.074, 0.9181], solución factible
- ind 4: [0.0515, -0.0759, 0.9181], solución factible

Reparación de soluciones:

- ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851
- ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851
- ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851
- ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:

- ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851

PSA: Ejemplo práctico - resultado final

Soluciones obtenidas:

ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851

ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851

ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851

ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:

ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851