Pendulum Search Algorithm

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Pendulum Search Algorithm (PSA)

- Fue desarrollada por Nor Azlina Ab. Aziz et. al. en el año 2022 [a].
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoreamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

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[a] *Pendulum Search Algorithm: An Optimization Algorithm Based on Simple Harmonic Motion and Its Application for a Vaccine Distribution Problem*, Algorithms (2022)

PSA: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t}\right) \quad (1)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right) \quad (2)$$

- Donde:
  - $X_{i,j}^{t}$ es la posición de la i-ésima solución en la j-ésima dimensión y en la t-ésima iteración
  - $Best_{j}^{t}$ es la mejor solución en la j-ésima dimensión y en la t-ésima iteración
  - $pend_{i,j}^{t}$, es un parámetro donde:
    - t es la iteración actual
    - t_{max} es el número total de iteraciones
    - $rand$ es un número aleatorio entre $[0, 1]$

PSA: Pseudocódigo

Algorithm 1 Pendulum Search Algorithm

Input: The population $X = \{X_1, X_2, ..., X_m\}$
Output: The updated population $X' = \{X'_1, X'_2, ..., X'_m\}$ and $Best^t$

Initialize random population X
Evaluate the solutions in the objective function
Update $Best^t$
for t = 1 to T **do**
  for i = 1 to m **do**
    for j = 1 to D **do** ▷ D = number of dimensions
      $pend^t_{i,j} = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$
      $X^{t+1}_{i,j} = X^t_{i,j} + pend^t_{i,j} \cdot \left(Best^t_j - X^t_{i,j}\right)$
    end for
  end for
  Evaluate the solutions in the objective function
  Update $Best^t$
end for
Return the updated population X' where *Best* is the best result

PSA: Ejemplo prático - parámetros iniciales

Considerando

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2$$

Sujeto a

$$x_1, x_2, x_3 \in [-100, 100]$$

Configuración inicial de PSA:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 100 iteraciones
- Constante a = 2

PSA: Ejemplo prático - soluciones inciales

Soluciones inciales:
ind 1: [-33.7939, 94.2896, 38.8342] / fitness: 11540.6514
ind 2: [53.5519, 7.0044, -91.4171] / fitness: 11273.9538
ind 3: [-93.6863, -78.8417, -19.722] / fitness: 15382.0938
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

PSA: Ejemplo práctico - iter 1 - ind 1 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,1}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}3094))$$
$$pend_{1,1}^{1} = -0{,}7292 \ / \ X_{1,1}^{1} = -33{,}7939 \ / \ Best_{1}^{1} = 24{,}8353$$

$$X_{1,1}^{2} = X_{1,1}^{1} + pend_{1,1}^{1} \cdot \left(Best_{1}^{1} - X_{1,1}^{1} \right)$$
$$X_{1,1}^{2} = -33{,}7939 - 0{,}7292 \cdot (24{,}8353 + 33{,}7939)$$
$$X_{1,1}^{2} = -76{,}5463$$

PSA: Ejemplo práctico - iter 1 - ind 1 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,2}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}0177))$$
$$pend_{1,2}^{1} = 1{,}9876 \ / \ X_{1,2}^{1} = 94{,}2896 \ / \ Best_{2}^{1} = -38{,}6385$$

$$X_{1,2}^{2} = X_{1,2}^{1} + pend_{1,2}^{1} \cdot \left(Best_{2}^{1} - X_{1,2}^{1} \right)$$
$$X_{1,2}^{2} = 94{,}2896 + 1{,}9876 \cdot (-38{,}6385 - 94{,}2896)$$
$$X_{1,2}^{2} = -169{,}9183$$

PSA: Ejemplo práctico - iter 1 - ind 1 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{1,3}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}0553))$
$pend_{1,3}^{1} = 1{,}8805$ / $X_{1,3}^{1} = 38{,}8342$ / $Best_{3}^{1} = 77{,}9742$

$X_{1,3}^{2} = X_{1,3}^{1} + pend_{1,3}^{1} \cdot \left(Best_{3}^{1} - X_{1,3}^{1} \right)$
$X_{1,3}^{2} = 38{,}8342 + 1{,}8805 \cdot (77{,}9742 - 38{,}8342)$
$X_{1,3}^{2} = 112{,}437$

PSA: Ejemplo práctico - iter 1 - ind 2 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{2,1}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}8257))$
$pend_{2,1}^{1} = 0{,}9158$ / $X_{2,1}^{1} = 53{,}5519$ / $Best_{1}^{1} = 24{,}8353$

$X_{2,1}^{2} = X_{2,1}^{1} + pend_{2,1}^{1} \cdot \left(Best_{1}^{1} - X_{2,1}^{1} \right)$
$X_{2,1}^{2} = 53{,}5519 + 0{,}9158 \cdot (24{,}8353 - 53{,}5519)$
$X_{2,1}^{2} = 27{,}2532$

PSA: Ejemplo práctico - iter 1 - ind 2 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,2}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}7772))$$
$$pend_{2,2}^{1} = 0{,}3401 \ / \ X_{2,2}^{1} = 7{,}0044 \ / \ Best_{2}^{1} = -38{,}6385$$

$$X_{2,2}^{2} = X_{2,2}^{1} + pend_{2,2}^{1} \cdot \left(Best_{2}^{1} - X_{2,2}^{1} \right)$$
$$X_{2,2}^{2} = 7{,}0044 + 0{,}3401 \cdot (-38{,}6385 - 7{,}0044)$$
$$X_{2,2}^{2} = -8{,}5188$$

PSA: Ejemplo práctico - iter 1 - ind 2 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{2,3}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4459))$
$pend_{2,3}^{1} = -1{,}8856 \ / \ X_{2,3}^{1} = -91{,}4171 \ / \ Best_{3}^{1} = 77{,}9742$

$X_{2,3}^{2} = X_{2,3}^{1} + pend_{2,3}^{1} \cdot \left(Best_{3}^{1} - X_{2,3}^{1} \right)$
$X_{2,3}^{2} = -91{,}4171 - 1{,}8856 \cdot (77{,}9742 + 91{,}4171)$
$X_{2,3}^{2} = -410{,}8213$

PSA: Ejemplo práctico - iter 1 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}6315))$$
$$pend_{3,1}^{1} = -1{,}3553 \ / \ X_{3,1}^{1} = -93{,}6863 \ / \ Best_{1}^{1} = 24{,}8353$$

$$X_{3,1}^{2} = X_{3,1}^{1} + pend_{3,1}^{1} \cdot \left(Best_{1}^{1} - X_{3,1}^{1} \right)$$
$$X_{3,1}^{2} = -93{,}6863 - 1{,}3553 \cdot (24{,}8353 + 93{,}6863)$$
$$X_{3,1}^{2} = -254{,}3186$$

PSA: Ejemplo práctico - iter 1 - ind 3 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{3,2}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}5219))$
$pend_{3,2}^{1} = -1{,}9811 \ / \ X_{3,2}^{1} = -78{,}8417 \ / \ Best_{2}^{1} = -38{,}6385$

$X_{3,2}^{2} = X_{3,2}^{1} + pend_{3,2}^{1} \cdot \left(Best_{2}^{1} - X_{3,2}^{1} \right)$
$X_{3,2}^{2} = -78{,}8417 - 1{,}9811 \cdot (-38{,}6385 + 78{,}8417)$
$X_{3,2}^{2} = -158{,}4883$

PSA: Ejemplo práctico - iter 1 - ind 3 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,3}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}2278))$$
$$pend_{3,3}^{1} = 0{,}2781 \ / \ X_{3,3}^{1} = -19{,}722 \ / \ Best_{3}^{1} = 77{,}9742$$

$$X_{3,3}^{2} = X_{3,3}^{1} + pend_{3,3}^{1} \cdot \left(Best_{3}^{1} - X_{3,3}^{1} \right)$$
$$X_{3,3}^{2} = -19{,}722 + 0{,}2781 \cdot (77{,}9742 + 19{,}722)$$
$$X_{3,3}^{2} = 7{,}4473$$

PSA: Ejemplo práctico - iter 1 - ind 4 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,1}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4123))$
$pend_{4,1}^{1} = -1{,}704 \ / \ X_{4,1}^{1} = 24{,}8353 \ / \ Best_{1}^{1} = 24{,}8353$

$X_{4,1}^{2} = X_{4,1}^{1} + pend_{4,1}^{1} \cdot \left(Best_{1}^{1} - X_{4,1}^{1} \right)$
$X_{4,1}^{2} = 24{,}8353 - 1{,}704 \cdot (24{,}8353 - 24{,}8353)$
$X_{4,1}^{2} = 24{,}8353$

PSA: Ejemplo práctico - iter 1 - ind 4 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,2}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}5089))$
$pend_{4,2}^{1} = -1{,}9969 \ / \ X_{4,2}^{1} = -38{,}6385 \ / \ Best_{2}^{1} = -38{,}6385$

$X_{4,2}^{2} = X_{4,2}^{1} + pend_{4,2}^{1} \cdot \left(Best_{2}^{1} - X_{4,2}^{1} \right)$
$X_{4,2}^{2} = -38{,}6385 - 1{,}9969 \cdot (-38{,}6385 + 38{,}6385)$
$X_{4,2}^{2} = -38{,}6385$

PSA: Ejemplo práctico - iter 1 - ind 4 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^{1} = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}9267))$$
$$pend_{4,3}^{1} = 1{,}7916 \ / \ X_{4,3}^{1} = 77{,}9742 \ / \ Best_{3}^{1} = 77{,}9742$$

$$X_{4,3}^{2} = X_{4,3}^{1} + pend_{4,3}^{1} \cdot \left(Best_{3}^{1} - X_{4,3}^{1} \right)$$
$$X_{4,3}^{2} = 77{,}9742 + 1{,}7916 \cdot (77{,}9742 - 77{,}9742)$$
$$X_{4,3}^{2} = 77{,}9742$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:
ind 1: [-76.5463, -**169.9183**, **112.437**], **dos dimensiones infactibles**
ind 2: [27.2532, -8.5188, -**410.8213**], **una dimensión infactible**
ind 3: [-**254.3186**, -**158.4883**, 7.4473], **dos dimensiones infactibles**
ind 4: [24.8353, -38.6385, 77.9742], solución factible

Reparación de soluciones:
ind 1: [-76.5463, -100.0, 100.0] / fitness: 25859.336
ind 2: [27.2532, -8.5188, -100.0] / fitness: 10815.3069
ind 3: [-100.0, -100.0, 7.4473] / fitness: 20055.4623
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

PSA: Ejemplo práctico - iter 2 - ind 1 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,1}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}7986))$$
$$pend_{1,1}^{2} = 0{,}5953 \ / \ X_{1,1}^{2} = -76{,}5463 \ / \ Best_{1}^{2} = 24{,}8353$$

$$X_{1,1}^{3} = X_{1,1}^{2} + pend_{1,1}^{2} \cdot \left(Best_{1}^{2} - X_{1,1}^{2} \right)$$
$$X_{1,1}^{3} = -76{,}5463 + 0{,}5953 \cdot (24{,}8353 + 76{,}5463)$$
$$X_{1,1}^{3} = -16{,}1938$$

PSA: Ejemplo práctico - iter 2 - ind 1 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{1,2}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4083))$
$pend_{1,2}^{2} = -1{,}6604 \ / \ X_{1,2}^{2} = -100{,}0 \ / \ Best_{2}^{2} = -38{,}6385$

$X_{1,2}^{3} = X_{1,2}^{2} + pend_{1,2}^{2} \cdot \left(Best_{2}^{2} - X_{1,2}^{2} \right)$
$X_{1,2}^{3} = -100{,}0 - 1{,}6604 \cdot (-38{,}6385 + 100{,}0)$
$X_{1,2}^{3} = -201{,}8846$

PSA: Ejemplo práctico - iter 2 - ind 1 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{1,3}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}867))$
$pend_{1,3}^{2} = 1{,}328 \ / \ X_{1,3}^{2} = 100{,}0 \ / \ Best_{3}^{2} = 77{,}9742$

$X_{1,3}^{3} = X_{1,3}^{2} + pend_{1,3}^{2} \cdot \left(Best_{3}^{2} - X_{1,3}^{2} \right)$
$X_{1,3}^{3} = 100{,}0 + 1{,}328 \cdot (77{,}9742 - 100{,}0)$
$X_{1,3}^{3} = 70{,}7497$

PSA: Ejemplo práctico - iter 2 - ind 2 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,1}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}7687))$$
$$pend_{2,1}^{2} = 0{,}2321 \ / \ X_{2,1}^{2} = 27{,}2532 \ / \ Best_{1}^{2} = 24{,}8353$$

$$X_{2,1}^{3} = X_{2,1}^{2} + pend_{2,1}^{2} \cdot \left(Best_{1}^{2} - X_{2,1}^{2} \right)$$
$$X_{2,1}^{3} = 27{,}2532 + 0{,}2321 \cdot (24{,}8353 - 27{,}2532)$$
$$X_{2,1}^{3} = 26{,}692$$

PSA: Ejemplo práctico - iter 2 - ind 2 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{2,2}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}323))$
$pend_{2,2}^{2} = -0{,}8767 \ / \ X_{2,2}^{2} = -8{,}5188 \ / \ Best_{2}^{2} = -38{,}6385$

$X_{2,2}^{3} = X_{2,2}^{2} + pend_{2,2}^{2} \cdot \left(Best_{2}^{2} - X_{2,2}^{2} \right)$
$X_{2,2}^{3} = -8{,}5188 - 0{,}8767 \cdot (-38{,}6385 + 8{,}5188)$
$X_{2,2}^{3} = 17{,}8871$

PSA: Ejemplo práctico - iter 2 - ind 2 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,3}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}5507))$$
$$pend_{2,3}^{2} = -1{,}8805 \ / \ X_{2,3}^{2} = -100{,}0 \ / \ Best_{3}^{2} = 77{,}9742$$

$$X_{2,3}^{3} = X_{2,3}^{2} + pend_{2,3}^{2} \cdot \left(Best_{3}^{2} - X_{2,3}^{2} \right)$$
$$X_{2,3}^{3} = -100{,}0 - 1{,}8805 \cdot (77{,}9742 + 100{,}0)$$
$$X_{2,3}^{3} = -434{,}6805$$

PSA: Ejemplo práctico - iter 2 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}1147))$$
$$pend_{3,1}^{2} = 1{,}4878 \ / \ X_{3,1}^{2} = -100{,}0 \ / \ Best_{1}^{2} = 24{,}8353$$

$$X_{3,1}^{3} = X_{3,1}^{2} + pend_{3,1}^{2} \cdot \left(Best_{1}^{2} - X_{3,1}^{2} \right)$$
$$X_{3,1}^{3} = -100{,}0 + 1{,}4878 \cdot (24{,}8353 + 100{,}0)$$
$$X_{3,1}^{3} = 85{,}73$$

PSA: Ejemplo práctico - iter 2 - ind 3 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{3,2}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}7433))$
$pend_{3,2}^{2} = -0{,}0833 \ / \ X_{3,2}^{2} = -100{,}0 \ / \ Best_{2}^{2} = -38{,}6385$

$X_{3,2}^{3} = X_{3,2}^{2} + pend_{3,2}^{2} \cdot \left(Best_{2}^{2} - X_{3,2}^{2} \right)$
$X_{3,2}^{3} = -100{,}0 - 0{,}0833 \cdot (-38{,}6385 + 100{,}0)$
$X_{3,2}^{3} = -105{,}1114$

PSA: Ejemplo práctico - iter 2 - ind 3 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{3,3}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}6752))$
$pend_{3,3}^{2} = -0{,}8967 \ / \ X_{3,3}^{2} = 7{,}4473 \ / \ Best_{3}^{2} = 77{,}9742$

$X_{3,3}^{3} = X_{3,3}^{2} + pend_{3,3}^{2} \cdot \left(Best_{3}^{2} - X_{3,3}^{2} \right)$
$X_{3,3}^{3} = 7{,}4473 - 0{,}8967 \cdot (77{,}9742 - 7{,}4473)$
$X_{3,3}^{3} = -55{,}7942$

PSA: Ejemplo práctico - iter 2 - ind 4 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,1}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}2914))$
$pend_{4,1}^{2} = -0{,}5093 \ / \ X_{4,1}^{2} = 24{,}8353 \ / \ Best_{1}^{2} = 24{,}8353$

$X_{4,1}^{3} = X_{4,1}^{2} + pend_{4,1}^{2} \cdot \left(Best_{1}^{2} - X_{4,1}^{2} \right)$
$X_{4,1}^{3} = 24{,}8353 - 0{,}5093 \cdot (24{,}8353 - 24{,}8353)$
$X_{4,1}^{3} = 24{,}8353$

PSA: Ejemplo práctico - iter 2 - ind 4 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,2}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}3383))$
$pend_{4,2}^{2} = -1{,}0431 \ / \ X_{4,2}^{2} = -38{,}6385 \ / \ Best_{2}^{2} = -38{,}6385$

$X_{4,2}^{3} = X_{4,2}^{2} + pend_{4,2}^{2} \cdot \left(Best_{2}^{2} - X_{4,2}^{2} \right)$
$X_{4,2}^{3} = -38{,}6385 - 1{,}0431 \cdot (-38{,}6385 + 38{,}6385)$
$X_{4,2}^{3} = -38{,}6385$

PSA: Ejemplo práctico - iter 2 - ind 4 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^{2} = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}1651))$$
$$pend_{4,3}^{2} = 1{,}0069 \ / \ X_{4,3}^{2} = 77{,}9742 \ / \ Best_{3}^{2} = 77{,}9742$$

$$X_{4,3}^{3} = X_{4,3}^{2} + pend_{4,3}^{2} \cdot \left(Best_{3}^{2} - X_{4,3}^{2} \right)$$
$$X_{4,3}^{3} = 77{,}9742 + 1{,}0069 \cdot (77{,}9742 - 77{,}9742)$$
$$X_{4,3}^{3} = 77{,}9742$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:
ind 1: [-16.1938, -**201.8846**, 70.7497], **una dimensión factible**
ind 2: [26.692, 17.8871, -**434.6805**], **una dimensión factible**
ind 3: [85.73, -**105.1114**, -55.7942], **una dimensión factible**
ind 4: [24.8353, -38.6385, 77.9742], solución factible

Reparación de soluciones:
ind 1: [-16.1938, -100.0, 70.7497] / fitness: 15267.7592
ind 2: [26.692, 17.8871, -100.0] / fitness: 11032.4112
ind 3: [85.73, -100.0, -55.7942] / fitness: 20462.6257
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

PSA: Ejemplo práctico - iter 100 - ind 1 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{1,1}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}6355))$
$pend_{1,1}^{100} = -0{,}4897 \ / \ X_{1,1}^{100} = 0{,}0515 \ / \ Best_{1}^{100} = 0{,}0515$

$X_{1,1}^{101} = X_{1,1}^{100} + pend_{1,1}^{100} \cdot \left(Best_{1}^{100} - X_{1,1}^{100} \right)$
$X_{1,1}^{101} = 0{,}0515 - 0{,}4897 \cdot (0{,}0515 - 0{,}0515)$
$X_{1,1}^{101} = 0{,}0515$

PSA: Ejemplo práctico - iter 100 - ind 1 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,2}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4321))$$
$$pend_{1,2}^{100} = -0{,}6765 \ / \ X_{1,2}^{100} = -0{,}074 \ / \ Best_{2}^{100} = -0{,}074$$

$$X_{1,2}^{101} = X_{1,2}^{100} + pend_{1,2}^{100} \cdot \left(Best_{2}^{100} - X_{1,2}^{100} \right)$$
$$X_{1,2}^{101} = -0{,}074 - 0{,}6765 \cdot (-0{,}074 + 0{,}074)$$
$$X_{1,2}^{101} = -0{,}074$$

PSA: Ejemplo práctico - iter 100 - ind 1 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{1,3}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}5042))$$
$$pend_{1,3}^{100} = -0{,}7429 \ / \ X_{1,3}^{100} = 0{,}9181 \ / \ Best_{3}^{100} = 0{,}9181$$

$$X_{1,3}^{101} = X_{1,3}^{100} + pend_{1,3}^{100} \cdot \left(Best_{3}^{100} - X_{1,3}^{100} \right)$$
$$X_{1,3}^{101} = 0{,}9181 - 0{,}7429 \cdot (0{,}9181 - 0{,}9181)$$
$$X_{1,3}^{101} = 0{,}9181$$

PSA: Ejemplo práctico - iter 100 - ind 2 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,1}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}3462))$$
$$pend_{2,1}^{100} = -0{,}4223 \ / \ X_{2,1}^{100} = 0{,}0515 \ / \ Best_{1}^{100} = 0{,}0515$$

$$X_{2,1}^{101} = X_{2,1}^{100} + pend_{2,1}^{100} \cdot \left(Best_{1}^{100} - X_{2,1}^{100} \right)$$
$$X_{2,1}^{101} = 0{,}0515 - 0{,}4223 \cdot (0{,}0515 - 0{,}0515)$$
$$X_{2,1}^{101} = 0{,}0515$$

PSA: Ejemplo práctico - iter 100 - ind 2 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,2}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}3179))$$
$$pend_{2,2}^{100} = -0{,}3075 \ / \ X_{2,2}^{100} = -0{,}074 \ / \ Best_{2}^{100} = -0{,}074$$

$$X_{2,2}^{101} = X_{2,2}^{100} + pend_{2,2}^{100} \cdot \left(Best_{2}^{100} - X_{2,2}^{100} \right)$$
$$X_{2,2}^{101} = -0{,}074 - 0{,}3075 \cdot (-0{,}074 + 0{,}074)$$
$$X_{2,2}^{101} = -0{,}074$$

PSA: Ejemplo práctico - iter 100 - ind 2 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{2,3}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}0765))$$
$$pend_{2,3}^{100} = 0{,}6589 \ / \ X_{2,3}^{100} = 0{,}9181 \ / \ Best_{3}^{100} = 0{,}9181$$

$$X_{2,3}^{101} = X_{2,3}^{100} + pend_{2,3}^{100} \cdot \left(Best_{3}^{100} - X_{2,3}^{100} \right)$$
$$X_{2,3}^{101} = 0{,}9181 + 0{,}6589 \cdot (0{,}9181 - 0{,}9181)$$
$$X_{2,3}^{101} = 0{,}9181$$

PSA: Ejemplo práctico - iter 100 - ind 3 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,1}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4507))$$
$$pend_{3,1}^{100} = -0{,}7078 \ / \ X_{3,1}^{100} = 0{,}0515 \ / \ Best_{1}^{100} = 0{,}0515$$

$$X_{3,1}^{101} = X_{3,1}^{100} + pend_{3,1}^{100} \cdot \left(Best_{1}^{100} - X_{3,1}^{100} \right)$$
$$X_{3,1}^{101} = 0{,}0515 - 0{,}7078 \cdot (0{,}0515 - 0{,}0515)$$
$$X_{3,1}^{101} = 0{,}0515$$

PSA: Ejemplo práctico - iter 100 - ind 3 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,2}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}741))$$
$$pend_{3,2}^{100} = -0{,}042 \ / \ X_{3,2}^{100} = -0{,}074 \ / \ Best_{2}^{100} = -0{,}074$$

$$X_{3,2}^{101} = X_{3,2}^{100} + pend_{3,2}^{100} \cdot \left(Best_{2}^{100} - X_{3,2}^{100} \right)$$
$$X_{3,2}^{101} = -0{,}074 - 0{,}042 \cdot (-0{,}074 + 0{,}074)$$
$$X_{3,2}^{101} = -0{,}074$$

PSA: Ejemplo práctico - iter 100 - ind 3 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{3,3}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}5312))$$
$$pend_{3,3}^{100} = -0{,}7289 \ / \ X_{3,3}^{100} = 0{,}9181 \ / \ Best_{3}^{100} = 0{,}9181$$

$$X_{3,3}^{101} = X_{3,3}^{100} + pend_{3,3}^{100} \cdot \left(Best_{3}^{100} - X_{3,3}^{100} \right)$$
$$X_{3,3}^{101} = 0{,}9181 - 0{,}7289 \cdot (0{,}9181 - 0{,}9181)$$
$$X_{3,3}^{101} = 0{,}9181$$

PSA: Ejemplo práctico - iter 100 - ind 4 - dim 1

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,1}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}8716))$$
$$pend_{4,1}^{100} = 0{,}5141 \ / \ X_{4,1}^{100} = 0{,}0515 \ / \ Best_{1}^{100} = 0{,}0515$$

$$X_{4,1}^{101} = X_{4,1}^{100} + pend_{4,1}^{100} \cdot \left(Best_{1}^{100} - X_{4,1}^{100} \right)$$
$$X_{4,1}^{101} = 0{,}0515 + 0{,}5141 \cdot (0{,}0515 - 0{,}0515)$$
$$X_{4,1}^{101} = 0{,}0515$$

PSA: Ejemplo práctico - iter 100 - ind 4 - dim 2

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,2}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}6165))$$
$$pend_{4,2}^{100} = -0{,}5528 \ / \ X_{4,2}^{100} = -0{,}0752 \ / \ Best_{2}^{100} = -0{,}074$$

$$X_{4,2}^{101} = X_{4,2}^{100} + pend_{4,2}^{100} \cdot \left(Best_{2}^{100} - X_{4,2}^{100} \right)$$
$$X_{4,2}^{101} = -0{,}0752 - 0{,}5528 \cdot (-0{,}074 + 0{,}0752)$$
$$X_{4,2}^{101} = -0{,}0759$$

PSA: Ejemplo práctico - iter 100 - ind 4 - dim 3

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{t_{max}}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$$pend_{4,3}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}3904))$$
$$pend_{4,3}^{100} = -0{,}5738 \ / \ X_{4,3}^{100} = 0{,}9181 \ / \ Best_{3}^{100} = 0{,}9181$$

$$X_{4,3}^{101} = X_{4,3}^{100} + pend_{4,3}^{100} \cdot \left(Best_{3}^{100} - X_{4,3}^{100} \right)$$
$$X_{4,3}^{101} = 0{,}9181 - 0{,}5738 \cdot (0{,}9181 - 0{,}9181)$$
$$X_{4,3}^{101} = 0{,}9181$$

PSA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 2:
ind 1: [0.0515, -0.074, 0.9181], solución factible
ind 2: [0.0515, -0.074, 0.9181], solución factible
ind 3: [0.0515, -0.074, 0.9181], solución factible
ind 4: [0.0515, -0.0759, 0.9181], solución factible

Reparación de soluciones:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851

PSA: Ejemplo práctico - resultado final

Soluciones obtenidas:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851