

Particle Swarm Optimization

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Particle Swarm Optimization (PSO)

- Fue desarrollada por Federico Marini et. al. en el año 2015 ^a.
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoriamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

^a *Particle swarm optimization (PSO). A tutorial*, Elsevier (2015)

- Ecuaciones de movimientos general

$$bestX = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix} \quad (1)$$

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,j} \\ x_{2,1} & x_{2,2} & \dots & x_{2,j} \\ \dots & \dots & \dots & \dots \\ x_{i,1} & x_{i,2} & \dots & x_{i,j} \end{bmatrix} \quad (2)$$

$$vMax = [-20, 20] \quad (3)$$

- Donde:
 - $vMax$ es un rango que limita la velocidad de las partículas

- Ecuaciones de movimientos general

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter} \quad (4)$$

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j}) \quad (5)$$

$$X_{i,j} = X_{i,j} + v_{i,j} \quad (6)$$

- Donde:

- $wMax = 1,0$
- $wMin = 0,2$
- $c_1 = 2$ y $c_2 = 2$

Algorithm 1 Particle Swarm Optimization

```
Input: Population  $X = \{x_1, x_2, \dots, x_N\}$   
Output: Updated population  $X' = \{x'_1, x'_2, \dots, x'_n\}$  and Best  
1: Initialize random particle population  $X$  and copy in BestX; constants: vMax, wMax, wMin, c1 and c2  
2: for it = 1 to MaxIt do  
3:     calculate the fitness of each  $X$   
4:     select Best  
5:     Check for each element in BestX if  $X[i]$  is better, then update BestX[i] accordingly  
6:      $v = \text{zeros}(N, \text{dim})$   
7:      $w = w\text{Max} - \text{it} * ((w\text{Max} - w\text{Min}) / \text{MaxIter})$   
8:     for i=0 to N do  
9:         for j=0 to dim do  
10:             $r1, r2 = \text{random}()$   
11:             $v[i,j] = w * v[i,j] + c1 * r1 * (\text{BestX}[i,j] - X[i,j]) + c2 * r2 * (\text{Best}[j] - X[i,j])$   
12:            Ensure  $v$  stays within the range  $[-v\text{Max}, v\text{Max}]$ ; if exceeded, adjust  $v$  to the nearest limit  
13:             $X[i,j] = X[i,j] + v[i,j]$   
14: return updated population  $X'$  and Best
```

Considerando

$$\text{Min } z = x_1^2 + x_2^2$$

Sujeto a

$$x_1, x_2 \in [-100, 100]$$

Configuración inicial de PSO:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 5000 iteraciones
- $v_{Max} = 20$
- $w_{Max} = 1,0$
- $w_{Min} = 0,2$
- $c_1 = 2$
- $c_2 = 2$

Soluciones iniciales:

ind 1: [65.5597, -89.2108] / fitness: 12256.6410

ind 2: [-68.0593, -9.6881] / fitness: 4725.9244

ind 3: [36.1261, 63.6346] / fitness: 5354.4598

ind 4: [-89.2303, 59.4155] / fitness: 11492.2387

Mejor solución:

ind 2: [-68.0593, -9.6881] / fitness: 4725.9244

PSO: Ejemplo práctico - iter 1

Ecuaciones generales de la iteración 1:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$
$$bestX = \begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,j} \\ X_{2,1} & X_{2,2} & \dots & X_{2,j} \\ \dots & \dots & \dots & \dots \\ X_{i,1} & X_{i,2} & \dots & X_{i,j} \end{bmatrix}$$

$$w = 1,0 - 1 \cdot \frac{1,0 - 0,2}{5000} = 0,9998$$
$$bestX = \begin{bmatrix} 65,5597 & -89,2108 \\ -68,0593 & -9,6881 \\ 36,1261 & 63,6346 \\ -89,2303 & 59,4155 \end{bmatrix}$$

PSO: Ejemplo práctico - iter 1 - ind 1 - dim 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,4119$$

$$r_2 = random(0, 1) = 0,1678$$

$$v_{1,1} = 0,9998 \cdot 0,0000 + 2 \cdot 0,4119 \cdot (65,5597 - 65,5597) + 2 \cdot 0,1678 \cdot (-68,0593 - 65,5597) = -44,8293$$

$$v_{1,1} < -v_{Max}$$

$$-44,8293 < -20$$

$$v_{1,1} = -v_{Max} = -20$$

$$X_{1,1} = 65,5597 - 20,0000 = 45,5597$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,1958$$

$$r_2 = random(0, 1) = 0,2073$$

$$v_{1,2} = 0,9998 \cdot 0,0000 + 2 \cdot 0,1958 \cdot (-89,2108 + 89,2108) + 2 \cdot 0,2073 \cdot (-9,6881 + 89,2108) = 32,9715$$

$$v_{1,2} > v_{Max}$$

$$32,9715 > 20$$

$$v_{1,2} = v_{Max} = 20$$

$$X_{1,2} = -89,2108 + 20,0000 = -69,2108$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,6211$$

$$r_2 = random(0, 1) = 0,2664$$

$$v_{2,1} = 0,9998 \cdot 0,0000 + 2 \cdot 0,6211 \cdot (-68,0593 + 68,0593) + 2 \cdot 0,2664 \cdot (-68,0593 + 68,0593) = 0,0000$$

$$X_{2,1} = -68,0593 + 0,0000 = -68,0593$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,1452$$

$$r_2 = random(0, 1) = 0,4729$$

$$v_{2,2} = 0,9998 \cdot 0,0000 + 2 \cdot 0,1452 \cdot (-9,6881 + 9,6881) + 2 \cdot 0,4729 \cdot (-9,6881 + 9,6881) = 0,0000$$

$$X_{2,2} = -9,6881 + 0,0000 = -9,6881$$

PSO: Ejemplo práctico - iter 1 - ind 3 - dim 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,4425$$

$$r_2 = random(0, 1) = 0,9511$$

$$v_{3,1} = 0,9998 \cdot 0,0000 + 2 \cdot 0,4425 \cdot (36,1261 - 36,1261) + 2 \cdot 0,9511 \cdot (-68,0593 - 36,1261) = -198,1876$$

$$v_{3,1} < -v_{Max}$$

$$-198,1876 < -20$$

$$v_{3,1} = v_{Max} = -20$$

$$X_{3,1} = 36,1261 + -20,0000 = 16,1261$$

PSO: Ejemplo práctico - iter 1 - ind 3 - dim 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,7923$$

$$r_2 = random(0, 1) = 0,2359$$

$$v_{1,1} = 0,9998 \cdot 0,0000 + 2 \cdot 0,7923 \cdot (63,6346 - 63,6346) + 2 \cdot 0,2359 \cdot (-9,6881 - 63,6346) = -34,5917$$

$$v_{3,2} < -v_{Max}$$

$$-34,5917 < -20$$

$$v_{3,2} = v_{Max} = -20$$

$$X_{3,2} = 63,6346 + -20,0000 = 43,6346$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = \text{random}(0, 1) = 0,7301$$

$$r_2 = \text{random}(0, 1) = 0,1236$$

$$v_{4,1} = 0,9998 \cdot 0,0000 + 2 \cdot 0,7301 \cdot (-89,2303 + 89,2303) + 2 \cdot 0,1236 \cdot (-68,0593 + 89,2303) = 5,2318$$

$$X_{4,1} = -89,2303 + 5,2318 = -83,9985$$

PSO: Ejemplo práctico - iter 1 - ind 4 - dim 2

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,2531$$

$$r_2 = random(0, 1) = 0,3419$$

$$v_{4,2} = 0,9998 \cdot 0,0000 + 2 \cdot 0,2531 \cdot (59,4155 - 59,4155) + 2 \cdot 0,3419 \cdot (-9,6881 - 59,4155) = -47,2525$$

$$v_{4,2} < -v_{Max}$$

$$-47,2525 < -20$$

$$v_{4,2} = v_{Max} = -20$$

$$X_{4,2} = 59,4155 + -20,0000 = 39,4155$$

Restricción: $x_1, x_2 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:

ind 1: [45.5597, -69.2108], infactibles: 0

ind 2: [-68.0593, -9.6881], infactibles: 0

ind 3: [16.1261, 43.6346], infactibles: 0

ind 4: [-83.9985, 39.4155], infactibles: 0

Reparación de soluciones:

ind 1: [45.5597, -69.2108] / fitness: 6865.8210

ind 2: [-68.0593, -9.6881] / fitness: 4725.9244

ind 3: [16.1261, 43.6346] / fitness: 2164.0316

ind 4: [-83.9985, 39.4155] / fitness: 8609.3302

Mejor solución:

ind 3: [16.1261, 43.6346] / fitness: 2164.0316

Ecuaciones generales de la iteración 2:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$
$$bestX = \begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,j} \\ X_{2,1} & X_{2,2} & \dots & X_{2,j} \\ \dots & \dots & \dots & \dots \\ X_{i,1} & X_{i,2} & \dots & X_{i,j} \end{bmatrix}$$

$$w = 1,0 - 2 \cdot \frac{1,0 - 0,2}{5000} = 0,9997$$
$$bestX = \begin{bmatrix} 45,5597 & -69,2108 \\ -68,0593 & -9,6881 \\ 16,1261 & 43,6346 \\ -83,9985 & 39,4155 \end{bmatrix}$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,1398$$

$$r_2 = random(0, 1) = 0,1348$$

$$v_{1,1} = 0,9997 \cdot 0,0000 + 2 \cdot 0,1398 \cdot (45,5597 - 45,5597) + 2 \cdot 0,1348 \cdot (16,1261 - 45,5597) = -7,9363$$

$$X_{1,1} = 45,5597 + -7,9363 = 37,6234$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,7543$$

$$r_2 = random(0, 1) = 0,7844$$

$$v_{1,2} = 0,9997 \cdot 0,0000 + 2 \cdot 0,7543 \cdot (-69,2108 + 69,2108) + 2 \cdot 0,7844 \cdot (43,6346 + 69,2108) = 177,0244$$

$$v_{1,2} > v_{Max}$$

$$177,0244 > 20$$

$$v_{1,2} = v_{Max} = 20$$

$$X_{1,2} = -69,2108 + 20,0000 = -49,2108$$

PSO: Ejemplo práctico - iter 2 - ind 2 - dim 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,3527$$

$$r_2 = random(0, 1) = 0,9777$$

$$v_{2,1} = 0,9997 \cdot 0,0000 + 2 \cdot 0,3527 \cdot (-68,0593 + 68,0593) + 2 \cdot 0,9777 \cdot (16,1261 + 68,0593) = 164,6212$$

$$v_{2,1} > v_{Max}$$

$$164,6212 > 20$$

$$v_{2,1} = v_{Max} = 20$$

$$X_{2,1} = -68,0593 + 20,0000 = -48,0593$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,6600$$

$$r_2 = random(0, 1) = 0,3774$$

$$v_{2,2} = 0,9997 \cdot 0,0000 + 2 \cdot 0,6600 \cdot (-9,6881 + 9,6881) + 2 \cdot 0,3774 \cdot (43,6346 + 9,6881) = 40,2440$$

$$v_{2,2} > v_{Max}$$

$$40,2440 > 20$$

$$v_{2,2} = v_{Max} = 20$$

$$X_{2,2} = -9,6881 + 20,0000 = 10,3119$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,3652$$

$$r_2 = random(0, 1) = 0,0702$$

$$v_{3,1} = 0,9997 \cdot 0,0000 + 2 \cdot 0,3652 \cdot (16,1261 - 16,1261) + 2 \cdot 0,0702 \cdot (16,1261 - 16,1261) = 0,0000$$

$$X_{3,1} = 16,1261 + 0,0000 = 16,1261$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,4273$$

$$r_2 = random(0, 1) = 0,8131$$

$$v_{3,2} = 0,9997 \cdot 0,0000 + 2 \cdot 0,4273 \cdot (43,6346 - 43,6346) + 2 \cdot 0,8131 \cdot (43,6346 - 43,6346) = 0,0000$$

$$X_{3,2} = 43,6346 + 0,0000 = 43,6346$$

PSO: Ejemplo práctico - iter 2 - ind 4 - dim 1

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,0994$$

$$r_2 = random(0, 1) = 0,2623$$

$$v_{4,1} = 0,9997 \cdot 0,0000 + 2 \cdot 0,0994 \cdot (-83,9985 + 83,9985) + 2 \cdot 0,2623 \cdot (16,1261 + 83,9985) = 52,5290$$

$$v_{4,1} > v_{Max}$$

$$52,5290 > 20$$

$$v_{4,1} = v_{Max} = 20$$

$$X_{4,1} = -83,9985 + 20,0000 = -63,9985$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,4001$$

$$r_2 = random(0, 1) = 0,5388$$

$$v_{4,2} = 0,9997 \cdot 0,0000 + 2 \cdot 0,4001 \cdot (39,4155 - 39,4155) + 2 \cdot 0,5388 \cdot (43,6346 - 39,4155) = 4,5466$$

$$X_{4,2} = 39,4155 + 4,5466 = 43,9620$$

PSO: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soluciones obtenidas en la iteración 2:

ind 1: [37.6234, -49.2108], infactibles: 0

ind 2: [-48.0593, 10.3119], infactibles: 0

ind 3: [16.1261, 43.6346], infactibles: 0

ind 4: [-63.9985, 43.962], infactibles: 0

Reparación de soluciones:

ind 1: [37.6234, -49.2108] / fitness: 3837.2256

ind 2: [-48.0593, 10.3119] / fitness: 2416.0277

ind 3: [16.1261, 43.6346] / fitness: 2164.0316

ind 4: [-63.9985, 43.962] / fitness: 6028.4711

Mejor solución:

ind 3: [16.1261, 43.6346] / fitness: 2164.0316

Ecuaciones generales de la iteración 5000:

$$w = w_{max} - iter \cdot \frac{w_{max} - w_{min}}{MaxIter}$$
$$bestX = \begin{bmatrix} X_{1,1} & X_{1,2} & \dots & X_{1,j} \\ X_{2,1} & X_{2,2} & \dots & X_{2,j} \\ \dots & \dots & \dots & \dots \\ X_{i,1} & X_{i,2} & \dots & X_{i,j} \end{bmatrix}$$

$$w = 1,0 - 5000 \cdot \frac{1,0 - 0,2}{5000} = 0,2000$$
$$bestX = \begin{bmatrix} -0,3212 & 0,0208 \\ -0,3212 & 0,0208 \\ -0,3212 & 0,0208 \\ -0,3212 & 0,0208 \end{bmatrix}$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,8887$$

$$r_2 = random(0, 1) = 0,5080$$

$$v_{1,1} = 0,2000 \cdot 0,0000 + 2 \cdot 0,8887 \cdot (-0,3212 + 0,3212) + 2 \cdot 0,5080 \cdot (-0,3212 + 0,3212) = 0,0000$$

$$X_{1,1} = -0,3212 + 0,0000 = -0,3212$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,1081$$

$$r_2 = random(0, 1) = 0,2942$$

$$v_{1,2} = 0,2000 \cdot 0,0000 + 2 \cdot 0,1081 \cdot (0,0208 - 0,0208) + 2 \cdot 0,2942 \cdot (0,0208 - 0,0208) = 0,0000$$

$$X_{1,2} = 0,0208 + 0,0000 = 0,0208$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,1395$$

$$r_2 = random(0, 1) = 0,1656$$

$$v_{2,1} = 0,2000 \cdot 0,0000 + 2 \cdot 0,1395 \cdot (-0,3212 + 0,3212) + 2 \cdot 0,1656 \cdot (-0,3212 + 0,3212) = 0,0000$$

$$X_{2,1} = -0,3212 + 0,0000 = -0,3212$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,6422$$

$$r_2 = random(0, 1) = 0,0291$$

$$v_{2,2} = 0,2000 \cdot 0,0000 + 2 \cdot 0,6422 \cdot (0,0208 - 0,0208) + 2 \cdot 0,0291 \cdot (0,0208 - 0,0208) = 0,0000$$

$$X_{2,2} = 0,0208 + 0,0000 = 0,0208$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,4158$$

$$r_2 = random(0, 1) = 0,3688$$

$$v_{3,1} = 0,2000 \cdot 0,0000 + 2 \cdot 0,4158 \cdot (-0,3212 + 0,3212) + 2 \cdot 0,3688 \cdot (-0,3212 + 0,3212) = 0,0000$$

$$X_{3,1} = -0,3212 + 0,0000 = -0,3212$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,7118$$

$$r_2 = random(0, 1) = 0,2098$$

$$v_{3,2} = 0,2000 \cdot 0,0000 + 2 \cdot 0,7118 \cdot (0,0208 - 0,0208) + 2 \cdot 0,2098 \cdot (0,0208 - 0,0208) = 0,0000$$

$$X_{3,2} = 0,0208 + 0,0000 = 0,0208$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = random(0, 1) = 0,7971$$

$$r_2 = random(0, 1) = 0,4384$$

$$v_{4,1} = 0,2000 \cdot 0,0000 + 2 \cdot 0,7971 \cdot (-0,3212 + 0,3212) + 2 \cdot 0,4384 \cdot (-0,3212 + 0,3212) = 0,0000$$

$$X_{4,1} = -0,3212 + 0,0000 = -0,3212$$

Ecuación general PSO:

$$v_{i,j} = w \cdot v_{i,j} + c_1 \cdot r_1 \cdot (BestX_{i,j} - X_{i,j}) + c_2 \cdot r_2 \cdot (BestX_j - X_{i,j})$$
$$X_{i,j} = X_{i,j} + v_{i,j}$$

$$r_1 = \text{random}(0, 1) = 0,5305$$

$$r_2 = \text{random}(0, 1) = 0,0261$$

$$v_{4,2} = 0,2000 \cdot 0,0000 + 2 \cdot 0,5305 \cdot (0,0208 - 0,0208) + 2 \cdot 0,0261 \cdot (0,0208 - 0,0208) = 0,0000$$

$$X_{4,2} = 0,0208 + 0,0000 = 0,0208$$

PSO: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2 \in [-100, 100]$

Soluciones obtenidas en la iteración 5000:

ind 1: [-0.3212, 0.0208], infactibles: 0

ind 2: [-0.3212, 0.0208], infactibles: 0

ind 3: [-0.3212, 0.0208], infactibles: 0

ind 4: [-0.3212, 0.0208], infactibles: 0

Reparación de soluciones:

ind 1: [-0.3212, 0.0208] / fitness: 0.1036

ind 2: [-0.3212, 0.0208] / fitness: 0.1036

ind 3: [-0.3212, 0.0208] / fitness: 0.1036

ind 4: [-0.3212, 0.0208] / fitness: 0.1036

Mejor solución:

ind 1: [-0.3212, 0.0208] / fitness: 0.1036