Pendulum Search Algorithm

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Pendulum Search Algorithm (PSA)

- Fue desarrollada por Nor Azlina Ab. Aziz et. al. en el año 2022 [a].
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoreamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

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[a] *Pendulum Search Algorithm: An Optimization Algorithm Based on Simple Harmonic Motion and Its Application for a Vaccine Distribution Problem*, Algorithms (2022)

PSA: Ecuaciones de movimiento

- Ecuaciones de movimientos general

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right) \tag{1}$$

$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right) \tag{2}$$

- Donde:
  - $X_{i,j}^t$ es la posición de la i-ésima solución en la j-ésima dimensión y en la t-ésima iteración
  - $Best_j^t$ es la mejor solución en la j-ésima dimensión y en la t-ésima iteración
  - $pend_{i,j}^t$, es un parámetro donde:
    - t es la iteración actual
    - $tmax$ es el número total de iteraciones
    - $rand$ es un número aleatorio entre $[0,1]$

PSA: Pseudocódigo

Algorithm 1 Pendulum Search Algorithm

    Input: The population $X = \{X_1, X_2, ..., X_m\}$
    Output: The updated population $X' = \{X'_1, X'_2, ..., X'_m\}$ and $Best^t$

1: Initialize random population X
2: Evaluate the solutions in the objective function
3: Update $Best^t$
4: **for** t = 1 to T **do**
5:     **for** i = 1 to m **do**
6:         **for** j = 1 to D **do**   ▷ D = number of dimensions
7:             $pend^t_{i,j} = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$
8:             $X^{t+1}_{i,j} = X^t_{i,j} + pend^t_{i,j} \cdot (Best^t_j - X^t_{i,j})$
9:         **end for**
10:     **end for**
11:     Evaluate the solutions in the objective function
12:     Update $Best^t$
13: **end for**
14: Return the updated population X' where $Best$ is the best result

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Considerando

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2$$

Sujeto a

$$x_1, x_2, x_3 \in [-100, 100]$$

Configuración inicial de PSA:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 100 iteraciones
- Constante a $= 2$

PSA: Ejemplo prático - soluciones inciales

Soluciones inciales:
ind 1: [-33.7939, 94.2896, 38.8342] / fitness: 11540.6514
ind 2: [53.5519, 7.0044, -91.4171] / fitness: 11273.9538
ind 3: [-93.6863, -78.8417, -19.722] / fitness: 15382.0938
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t}\right)$$
$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{1,1}^{1} = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0,3094\right)\right)$
$pend_{1,1}^{1} = -0,7292 \ / \ X_{1,1}^{1} = -33,7939 \ / \ Best_{1}^{1} = 24,8353$

$X_{1,1}^{2} = X_{1,1}^{1} + pend_{1,1}^{1} \cdot \left(Best_{1}^{1} - X_{1,1}^{1}\right)$
$X_{1,1}^{2} = -33,7939 - 0,7292 \cdot \left(24,8353 + 33,7939\right)$
$X_{0}, 0^{1} = -76,5463$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t}\right)$$
$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{1,2}^{1} = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}0177\right)\right)$
$pend_{1,2}^{1} = 1{,}9876 \ / \ X_{1,2}^{1} = 94{,}2896 \ / \ Best_{2}^{1} = -38{,}6385$

$X_{1,2}^{2} = X_{1,2}^{1} + pend_{1,2}^{1} \cdot \left(Best_{2}^{1} - X_{1,2}^{1}\right)$
$X_{1,2}^{2} = 94{,}2896 + 1{,}9876 \cdot \left(-38{,}6385 - 94{,}2896\right)$
$X_{0},1^{1} = -169{,}9183$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos{(2 \cdot \pi \cdot rand)})$$

$pend_{1,3}^1 = 2 \cdot exp{(-1/100)} \cdot (\cos{(2 \cdot \pi \cdot 0{,}0553)})$
$pend_{1,3}^1 = 1{,}8805 \ / \ X_{1,3}^1 = 38{,}8342 \ / \ Best_3^1 = 77{,}9742$

$X_{1,3}^2 = X_{1,3}^1 + pend_{1,3}^1 \cdot \left(Best_3^1 - X_{1,3}^1\right)$
$X_{1,3}^2 = 38{,}8342 + 1{,}8805 \cdot (77{,}9742 - 38{,}8342)$
$X_0, 2^1 = 112{,}437$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos{(2 \cdot \pi \cdot rand)})$$

$pend_{2,1}^1 = 2 \cdot exp{(-1/100)} \cdot (\cos{(2 \cdot \pi \cdot 0{,}8257)})$
$pend_{2,1}^1 = 0{,}9158 \ / \ X_{2,1}^1 = 53{,}5519 \ / \ Best_1^1 = 24{,}8353$

$X_{2,1}^2 = X_{2,1}^1 + pend_{2,1}^1 \cdot \left(Best_1^1 - X_{2,1}^1 \right)$
$X_{2,1}^2 = 53{,}5519 + 0{,}9158 \cdot (24{,}8353 - 53{,}5519)$
$X_1, 0^1 = 27{,}2532$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,2}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}7772\right)\right)$
$pend_{2,2}^1 = 0{,}3401 \ / \ X_{2,2}^1 = 7{,}0044 \ / \ Best_2^1 = -38{,}6385$

$X_{2,2}^2 = X_{2,2}^1 + pend_{2,2}^1 \cdot \left(Best_2^1 - X_{2,2}^1\right)$
$X_{2,2}^2 = 7{,}0044 + 0{,}3401 \cdot \left(-38{,}6385 - 7{,}0044\right)$
$X_1, 1^1 = -8{,}5188$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,3}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}4459\right)\right)$
$pend_{2,3}^1 = -1{,}8856 \ / \ X_{2,3}^1 = -91{,}4171 \ / \ Best_3^1 = 77{,}9742$

$X_{2,3}^2 = X_{2,3}^1 + pend_{2,3}^1 \cdot \left(Best_3^1 - X_{2,3}^1\right)$
$X_{2,3}^2 = -91{,}4171 - 1{,}8856 \cdot \left(77{,}9742 + 91{,}4171\right)$
$X_1, 2^1 = -410{,}8213$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{3,1}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}6315\right)\right)$
$pend_{3,1}^1 = -1{,}3553 \ / \ X_{3,1}^1 = -93{,}6863 \ / \ Best_1^1 = 24{,}8353$

$X_{3,1}^2 = X_{3,1}^1 + pend_{3,1}^1 \cdot \left(Best_1^1 - X_{3,1}^1\right)$
$X_{3,1}^2 = -93{,}6863 - 1{,}3553 \cdot \left(24{,}8353 + 93{,}6863\right)$
$X_2, 0^1 = -254{,}3186$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{3,2}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}5219\right)\right)$
$pend_{3,2}^1 = -1{,}9811 \ / \ X_{3,2}^1 = -78{,}8417 \ / \ Best_2^1 = -38{,}6385$

$X_{3,2}^2 = X_{3,2}^1 + pend_{3,2}^1 \cdot \left(Best_2^1 - X_{3,2}^1\right)$
$X_{3,2}^2 = -78{,}8417 - 1{,}9811 \cdot \left(-38{,}6385 + 78{,}8417\right)$
$X_2, 1^1 = -158{,}4883$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{3,3}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}2278\right)\right)$
$pend_{3,3}^1 = 0{,}2781 \ / \ X_{3,3}^1 = -19{,}722 \ / \ Best_3^1 = 77{,}9742$

$X_{3,3}^2 = X_{3,3}^1 + pend_{3,3}^1 \cdot \left(Best_3^1 - X_{3,3}^1\right)$
$X_{3,3}^2 = -19{,}722 + 0{,}2781 \cdot \left(77{,}9742 + 19{,}722\right)$
$X_2,2^1 = 7{,}4473$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,1}^1 = 2 \cdot exp(-1/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4123))$
$pend_{4,1}^1 = -1{,}704 \,/\, X_{4,1}^1 = 24{,}8353 \,/\, Best_1^1 = 24{,}8353$

$X_{4,1}^2 = X_{4,1}^1 + pend_{4,1}^1 \cdot \left(Best_1^1 - X_{4,1}^1\right)$
$X_{4,1}^2 = 24{,}8353 - 1{,}704 \cdot (24{,}8353 - 24{,}8353)$
$X_3, 0^1 = 24{,}8353$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{4,2}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}5089\right)\right)$
$pend_{4,2}^1 = -1{,}9969 \ / \ X_{4,2}^1 = -38{,}6385 \ / \ Best_2^1 = -38{,}6385$

$X_{4,2}^2 = X_{4,2}^1 + pend_{4,2}^1 \cdot \left(Best_2^1 - X_{4,2}^1\right)$
$X_{4,2}^2 = -38{,}6385 - 1{,}9969 \cdot \left(-38{,}6385 + 38{,}6385\right)$
$X_{3,}1^1 = -38{,}6385$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{4,3}^1 = 2 \cdot exp\left(-1/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}9267\right)\right)$
$pend_{4,3}^1 = 1{,}7916 \;/\; X_{4,3}^1 = 77{,}9742 \;/\; Best_3^1 = 77{,}9742$

$X_{4,3}^2 = X_{4,3}^1 + pend_{4,3}^1 \cdot \left(Best_3^1 - X_{4,3}^1\right)$
$X_{4,3}^2 = 77{,}9742 + 1{,}7916 \cdot (77{,}9742 - 77{,}9742)$
$X_3, 2^1 = 77{,}9742$

PSA: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:
ind 1: [-76.5463, -**169.9183**, **112.437**], **dos dimensiones infactibles**
ind 2: [27.2532, -8.5188, -**410.8213**], **una dimensión infactible**
ind 3: [-**254.3186**, -**158.4883**, 7.4473], **dos dimensiones infactibles**
ind 4: [24.8353, -38.6385, 77.9742], solución factible

Reparación de soluciones:
ind 1: [-76.5463, -100.0, 100.0] / fitness: 25859.336
ind 2: [27.2532, -8.5188, -100.0] / fitness: 10815.3069
ind 3: [-100.0, -100.0, 7.4473] / fitness: 20055.4623
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax} \right)} \cdot \left(\cos \left(2 \cdot \pi \cdot rand \right) \right)$$

$pend_{1,1}^2 = 2 \cdot exp \left(-2/100 \right) \cdot \left(\cos \left(2 \cdot \pi \cdot 0,7986 \right) \right)$
$pend_{1,1}^2 = 0,5953 \; / \; X_{1,1}^2 = -76,5463 \; / \; Best_1^2 = 24,8353$

$X_{1,1}^3 = X_{1,1}^2 + pend_{1,1}^2 \cdot \left(Best_1^2 - X_{1,1}^2 \right)$
$X_{1,1}^3 = -76,5463 + 0,5953 \cdot \left(24,8353 + 76,5463 \right)$
$X_0, 0^2 = -16,1938$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{1,2}^2 = 2 \cdot exp(-2/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}4083))$
$pend_{1,2}^2 = -1{,}6604 \ / \ X_{1,2}^2 = -100{,}0 \ / \ Best_2^2 = -38{,}6385$

$X_{1,2}^3 = X_{1,2}^2 + pend_{1,2}^2 \cdot \left(Best_2^2 - X_{1,2}^2\right)$
$X_{1,2}^3 = -100{,}0 - 1{,}6604 \cdot (-38{,}6385 + 100{,}0)$
$X_0, 1^2 = -201{,}8846$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{1,3}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}867\right)\right)$
$pend_{1,3}^2 = 1{,}328 \; / \; X_{1,3}^2 = 100{,}0 \; / \; Best_3^2 = 77{,}9742$

$X_{1,3}^3 = X_{1,3}^2 + pend_{1,3}^2 \cdot \left(Best_3^2 - X_{1,3}^2\right)$
$X_{1,3}^3 = 100{,}0 + 1{,}328 \cdot \left(77{,}9742 - 100{,}0\right)$
$X_0, 2^2 = 70{,}7497$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,1}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot {\color{red}0{,}7687}\right)\right)$
$pend_{2,1}^2 = 0{,}2321 \;/\; X_{2,1}^2 = 27{,}2532 \;/\; Best_1^2 = 24{,}8353$

$X_{2,1}^3 = X_{2,1}^2 + pend_{2,1}^2 \cdot \left(Best_1^2 - X_{2,1}^2\right)$
$X_{2,1}^3 = 27{,}2532 + 0{,}2321 \cdot \left(24{,}8353 - 27{,}2532\right)$
$X_1,0^2 = 26{,}692$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,2}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}323\right)\right)$
$pend_{2,2}^2 = -0{,}8767 \;/\; X_{2,2}^2 = -8{,}5188 \;/\; Best_2^2 = -38{,}6385$

$X_{2,2}^3 = X_{2,2}^2 + pend_{2,2}^2 \cdot \left(Best_2^2 - X_{2,2}^2\right)$
$X_{2,2}^3 = -8{,}5188 - 0{,}8767 \cdot \left(-38{,}6385 + 8{,}5188\right)$
$X_1,1^2 = 17{,}8871$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,3}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}5507\right)\right)$
$pend_{2,3}^2 = -1{,}8805$ / $X_{2,3}^2 = -100{,}0$ / $Best_3^2 = 77{,}9742$

$X_{2,3}^3 = X_{2,3}^2 + pend_{2,3}^2 \cdot \left(Best_3^2 - X_{2,3}^2\right)$
$X_{2,3}^3 = -100{,}0 - 1{,}8805 \cdot (77{,}9742 + 100{,}0)$
$X_1, 2^2 = -434{,}6805$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax} \right)} \cdot \left(\cos \left(2 \cdot \pi \cdot rand \right) \right)$$

$pend_{3,1}^2 = 2 \cdot exp \left(-2/100 \right) \cdot \left(\cos \left(2 \cdot \pi \cdot 0{,}1147 \right) \right)$
$pend_{3,1}^2 = 1{,}4878 \ / \ X_{3,1}^2 = -100{,}0 \ / \ Best_1^2 = 24{,}8353$

$X_{3,1}^3 = X_{3,1}^2 + pend_{3,1}^2 \cdot \left(Best_1^2 - X_{3,1}^2 \right)$
$X_{3,1}^3 = -100{,}0 + 1{,}4878 \cdot \left(24{,}8353 + 100{,}0 \right)$
$X_2, 0^2 = 85{,}73$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{3,2}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}7433\right)\right)$
$pend_{3,2}^2 = -0{,}0833 \ / \ X_{3,2}^2 = -100{,}0 \ / \ Best_2^2 = -38{,}6385$

$X_{3,2}^3 = X_{3,2}^2 + pend_{3,2}^2 \cdot \left(Best_2^2 - X_{3,2}^2\right)$
$X_{3,2}^3 = -100{,}0 - 0{,}0833 \cdot \left(-38{,}6385 + 100{,}0\right)$
$X_2, 1^2 = -105{,}1114$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos\left(2 \cdot \pi \cdot rand\right))$$

$pend_{3,3}^2 = 2 \cdot exp\left(-2/100\right) \cdot (\cos\left(2 \cdot \pi \cdot 0{,}6752\right))$
$pend_{3,3}^2 = -0{,}8967 \ / \ X_{3,3}^2 = 7{,}4473 \ / \ Best_3^2 = 77{,}9742$

$X_{3,3}^3 = X_{3,3}^2 + pend_{3,3}^2 \cdot \left(Best_3^2 - X_{3,3}^2\right)$
$X_{3,3}^3 = 7{,}4473 - 0{,}8967 \cdot (77{,}9742 - 7{,}4473)$
$X_2, 2^2 = -55{,}7942$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{4,1}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}2914\right)\right)$
$pend_{4,1}^2 = -0{,}5093 \;/\; X_{4,1}^2 = 24{,}8353 \;/\; Best_1^2 = 24{,}8353$

$X_{4,1}^3 = X_{4,1}^2 + pend_{4,1}^2 \cdot \left(Best_1^2 - X_{4,1}^2\right)$
$X_{4,1}^3 = 24{,}8353 - 0{,}5093 \cdot \left(24{,}8353 - 24{,}8353\right)$
$X_3, 0^2 = 24{,}8353$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{4,2}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}3383\right)\right)$
$pend_{4,2}^2 = -1{,}0431 \ / \ X_{4,2}^2 = -38{,}6385 \ / \ Best_2^2 = -38{,}6385$

$X_{4,2}^3 = X_{4,2}^2 + pend_{4,2}^2 \cdot \left(Best_2^2 - X_{4,2}^2\right)$
$X_{4,2}^3 = -38{,}6385 - 1{,}0431 \cdot \left(-38{,}6385 + 38{,}6385\right)$
$X_3, 1^2 = -38{,}6385$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{4,3}^2 = 2 \cdot exp\left(-2/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot \textcolor{red}{0,1651}\right)\right)$
$pend_{4,3}^2 = 1,0069 \ / \ X_{4,3}^2 = 77,9742 \ / \ Best_3^2 = 77,9742$

$X_{4,3}^3 = X_{4,3}^2 + pend_{4,3}^2 \cdot \left(Best_3^2 - X_{4,3}^2\right)$
$X_{4,3}^3 = 77,9742 + 1,0069 \cdot (77,9742 - 77,9742)$
$X_3, 2^2 = 77,9742$

PSA: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:
ind 1: [-16.1938, -**201.8846**, 70.7497], **una dimensión factible**
ind 2: [26.692, 17.8871, -**434.6805**], **una dimensión factible**
ind 3: [85.73, -**105.1114**, -55.7942], **una dimensión factible**
ind 4: [24.8353, -38.6385, 77.9742], solución factible

Reparación de soluciones:
ind 1: [-16.1938, -100.0, 70.7497] / fitness: 15267.7592
ind 2: [26.692, 17.8871, -100.0] / fitness: 11032.4112
ind 3: [85.73, -100.0, -55.7942] / fitness: 20462.6257
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Mejor solución:
ind 4: [24.8353, -38.6385, 77.9742] / fitness: 8189.7017

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{1,1}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}6355\right)\right)$
$pend_{1,1}^{100} = -0{,}4897 \ / \ X_{1,1}^{100} = 0{,}0515 \ / \ Best_1^{100} = 0{,}0515$

$X_{1,1}^{101} = X_{1,1}^{100} + pend_{1,1}^{100} \cdot \left(Best_1^{100} - X_{1,1}^{100} \right)$
$X_{1,1}^{101} = 0{,}0515 - 0{,}4897 \cdot \left(0{,}0515 - 0{,}0515\right)$
$X_0, 0^1 00 = 0{,}0515$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{1,2}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}4321\right)\right)$
$pend_{1,2}^{100} = -0{,}6765 \ / \ X_{1,2}^{100} = -0{,}074 \ / \ Best_2^{100} = -0{,}074$

$X_{1,2}^{101} = X_{1,2}^{100} + pend_{1,2}^{100} \cdot \left(Best_2^{100} - X_{1,2}^{100}\right)$
$X_{1,2}^{101} = -0{,}074 - 0{,}6765 \cdot \left(-0{,}074 + 0{,}074\right)$
$X_0, 1^1 00 = -0{,}074$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$

$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{tmax} \right)} \cdot \left(\cos \left(2 \cdot \pi \cdot rand \right) \right)$$

$pend_{1,3}^{100} = 2 \cdot exp\left(-100/100 \right) \cdot \left(\cos \left(2 \cdot \pi \cdot 0{,}5042 \right) \right)$

$pend_{1,3}^{100} = -0{,}7429$ / $X_{1,3}^{100} = 0{,}9181$ / $Best_{3}^{100} = 0{,}9181$

$X_{1,3}^{101} = X_{1,3}^{100} + pend_{1,3}^{100} \cdot \left(Best_{3}^{100} - X_{1,3}^{100} \right)$

$X_{1,3}^{101} = 0{,}9181 - 0{,}7429 \cdot \left(0{,}9181 - 0{,}9181 \right)$

$X_0, 2^1 00 = 0{,}9181$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,1}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}3462\right)\right)$
$pend_{2,1}^{100} = -0{,}4223 \; / \; X_{2,1}^{100} = 0{,}0515 \; / \; Best_1^{100} = 0{,}0515$

$X_{2,1}^{101} = X_{2,1}^{100} + pend_{2,1}^{100} \cdot \left(Best_1^{100} - X_{2,1}^{100}\right)$
$X_{2,1}^{101} = 0{,}0515 - 0{,}4223 \cdot \left(0{,}0515 - 0{,}0515\right)$
$X_1, 0^1 00 = 0{,}0515$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,2}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot \textcolor{red}{0{,}3179}\right)\right)$
$pend_{2,2}^{100} = -0{,}3075 \ / \ X_{2,2}^{100} = -0{,}074 \ / \ Best_2^{100} = -0{,}074$

$X_{2,2}^{101} = X_{2,2}^{100} + pend_{2,2}^{100} \cdot \left(Best_2^{100} - X_{2,2}^{100}\right)$
$X_{2,2}^{101} = -0{,}074 - 0{,}3075 \cdot \left(-0{,}074 + 0{,}074\right)$
$X_1, 1^1 00 = -0{,}074$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{2,3}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0,0765\right)\right)$
$pend_{2,3}^{100} = 0,6589 \ / \ X_{2,3}^{100} = 0,9181 \ / \ Best_3^{100} = 0,9181$

$X_{2,3}^{101} = X_{2,3}^{100} + pend_{2,3}^{100} \cdot \left(Best_3^{100} - X_{2,3}^{100}\right)$
$X_{2,3}^{101} = 0,9181 + 0,6589 \cdot \left(0,9181 - 0,9181\right)$
$X_1, 2^1 00 = 0,9181$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + pend_{i,j}^{t} \cdot \left(Best_{j}^{t} - X_{i,j}^{t} \right)$$
$$pend_{i,j}^{t} = 2 \cdot exp^{\left(\frac{-t}{tmax} \right)} \cdot \left(\cos \left(2 \cdot \pi \cdot rand \right) \right)$$

$pend_{3,1}^{100} = 2 \cdot exp \left(-100/100 \right) \cdot \left(\cos \left(2 \cdot \pi \cdot \textcolor{red}{0,4507} \right) \right)$
$pend_{3,1}^{100} = -0,7078 \ / \ X_{3,1}^{100} = 0,0515 \ / \ Best_{1}^{100} = 0,0515$

$X_{3,1}^{101} = X_{3,1}^{100} + pend_{3,1}^{100} \cdot \left(Best_{1}^{100} - X_{3,1}^{100} \right)$
$X_{3,1}^{101} = 0,0515 - 0,7078 \cdot \left(0,0515 - 0,0515 \right)$
$X_2, 0^1 00 = 0,0515$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right)\right)$$

$pend_{3,2}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}741\right)\right)$
$pend_{3,2}^{100} = -0{,}042 \ / \ X_{3,2}^{100} = -0{,}074 \ / \ Best_2^{100} = -0{,}074$

$X_{3,2}^{101} = X_{3,2}^{100} + pend_{3,2}^{100} \cdot \left(Best_2^{100} - X_{3,2}^{100}\right)$
$X_{3,2}^{101} = -0{,}074 - 0{,}042 \cdot \left(-0{,}074 + 0{,}074\right)$
$X_2, 1^100 = -0{,}074$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax} \right)} \cdot \left(\cos \left(2 \cdot \pi \cdot rand \right) \right)$$

$pend_{3,3}^{100} = 2 \cdot exp \left(-100/100 \right) \cdot \left(\cos \left(2 \cdot \pi \cdot 0{,}5312 \right) \right)$
$pend_{3,3}^{100} = -0{,}7289$ / $X_{3,3}^{100} = 0{,}9181$ / $Best_3^{100} = 0{,}9181$

$X_{3,3}^{101} = X_{3,3}^{100} + pend_{3,3}^{100} \cdot \left(Best_3^{100} - X_{3,3}^{100} \right)$
$X_{3,3}^{101} = 0{,}9181 - 0{,}7289 \cdot \left(0{,}9181 - 0{,}9181 \right)$
$X_2, 2^1 00 = 0{,}9181$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t \right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot \left(\cos\left(2 \cdot \pi \cdot rand\right) \right)$$

$pend_{4,1}^{100} = 2 \cdot exp\left(-100/100\right) \cdot \left(\cos\left(2 \cdot \pi \cdot 0{,}8716\right) \right)$
$pend_{4,1}^{100} = 0{,}5141 \: / \: X_{4,1}^{100} = 0{,}0515 \: / \: Best_1^{100} = 0{,}0515$

$X_{4,1}^{101} = X_{4,1}^{100} + pend_{4,1}^{100} \cdot \left(Best_1^{100} - X_{4,1}^{100} \right)$
$X_{4,1}^{101} = 0{,}0515 + 0{,}5141 \cdot \left(0{,}0515 - 0{,}0515 \right)$
$X_3, 0^1 00 = 0{,}0515$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,2}^{100} = 2 \cdot exp(-100/100) \cdot (\cos(2 \cdot \pi \cdot 0{,}6165))$
$pend_{4,2}^{100} = -0{,}5528 \;/\; X_{4,2}^{100} = -0{,}0752 \;/\; Best_2^{100} = -0{,}074$

$X_{4,2}^{101} = X_{4,2}^{100} + pend_{4,2}^{100} \cdot \left(Best_2^{100} - X_{4,2}^{100}\right)$
$X_{4,2}^{101} = -0{,}0752 - 0{,}5528 \cdot (-0{,}074 + 0{,}0752)$
$X_3, 1^1 00 = -0{,}0759$

Ecuación general PSA:

$$X_{i,j}^{t+1} = X_{i,j}^t + pend_{i,j}^t \cdot \left(Best_j^t - X_{i,j}^t\right)$$
$$pend_{i,j}^t = 2 \cdot exp^{\left(\frac{-t}{tmax}\right)} \cdot (\cos(2 \cdot \pi \cdot rand))$$

$pend_{4,3}^{100} = 2 \cdot exp\left(-100/100\right) \cdot (\cos(2 \cdot \pi \cdot \textcolor{red}{0{,}3904}))$
$pend_{4,3}^{100} = -0{,}5738 \ / \ X_{4,3}^{100} = 0{,}9181 \ / \ Best_3^{100} = 0{,}9181$

$X_{4,3}^{101} = X_{4,3}^{100} + pend_{4,3}^{100} \cdot \left(Best_3^{100} - X_{4,3}^{100}\right)$
$X_{4,3}^{101} = 0{,}9181 - 0{,}5738 \cdot (0{,}9181 - 0{,}9181)$
$X_3, 2^1 00 = 0{,}9181$

PSA: Ejemplo prático - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 2:
ind 1: [0.0515, -0.074, 0.9181], solución factible
ind 2: [0.0515, -0.074, 0.9181], solución factible
ind 3: [0.0515, -0.074, 0.9181], solución factible
ind 4: [0.0515, -0.0759, 0.9181], solución factible

Reparación de soluciones:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851

PSA: Ejemplo prático - resultado final

Soluciones obtenidas:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 2: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 3: [0.0515, -0.074, 0.9181] / fitness: 0.851
ind 4: [0.0515, -0.0759, 0.9181] / fitness: 0.8513

Mejor solución:
ind 1: [0.0515, -0.074, 0.9181] / fitness: 0.851