

Sine Cosine Algorithm

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Sine Cosine Algorithm (SCA)

- Fue desarrollada por Seyedali Mirjalili en el año 2015 ^a.
- Es una metaheurística basado en población diseñada para resolver problemas de optimización continuos.
- Sus soluciones (individuos) iniciales se generan aleatoriamente y se van alterando bajo un conjunto de reglas de movimiento con criterios estocásticos.

^aSCA: *A Sine Cosine Algorithm for Solving Optimization Problems*, Knowledge-Based Systems (2016)

- Ecuaciones de movimientos general

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases} \quad (1)$$

- Donde:

- $X_{i,j}^t$ es la posición de la i -ésima solución en la j -ésima dimensión y en la t -ésima iteración
- P_j^t es la mejor solución en la j -ésima dimensión y en la t -ésima iteración
- $r_1 = a - t \frac{a}{T}$, es un parámetro y a es una constante, t es la iteración actual y T es el número total de iteraciones
- $r_2 = 2 \cdot \pi \cdot \text{rand} [0, 1]$, es número aleatorio
- $r_3 = 2 \cdot \text{rand} [0, 1]$, es número aleatorio
- $r_4 = \text{rand} [0, 1]$, es número aleatorio

Algorithm 1 Sine Cosine Algorithm

Input: The population $X = \{X_1, X_2, \dots, X_m\}$
Output: The updated population $X' = \{X'_1, X'_2, \dots, X'_m\}$ and X_{best}

- 1: Initialize random population X
- 2: Set initial r_1
- 3: Evaluate the solutions in the objective function
- 4: Update X_{best}
- 5: **for** $t = 1$ to T **do**
- 6: **for** $i = 1$ to m **do**
- 7: **for** $d = 1$ to D **do** ▷ D = number of dimensions
- 8: Update P_d^t , where $P_d^t = X_{best,d}$
- 9: Randomly generate the value of r_2, r_3, r_4
- 10: Update the position of $X_{i,d}^t$ using Eq. (1)
- 11: **end for**
- 12: **end for**
- 13: Update r_1
- 14: Evaluate the solutions in the objective function
- 15: Update X_{best}
- 16: **end for**
- 17: Return the updated population X' where X_{best} is the best result

Considerando

$$\text{Min } z = x_1^2 + x_2^2 + x_3^2$$

Sujeto a

$$x_1, x_2, x_3 \in [-100, 100]$$

Configuración inicial de SCA:

- Tamaño de la población: 4 individuos
- Número máximo de iteraciones: 100 iteraciones
- Constante $a = 2$

Soluciones iniciales:

ind 1: [32.9303, 72.927, 44.8614] / fitness: 8415.2972

ind 2: [-11.202, 76.1347, 66.6762] / fitness: 10367.693

ind 3: [94.6737, -63.2213, 63.8176] / fitness: 17032.7283

ind 4: [77.6001, -94.4137, -11.8695] / fitness: 15076.6073

Mejor solución:

ind 1: [32.9303, 72.927, 44.8614] / fitness: 8415.2972

SCA: Ejemplo práctico - ind 1 dim 1 iter 1

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^1 = 32,9303 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,9482 = 5,9577 / r_3 = 2 \cdot 0,2531 = 0,5062$$

$$r_4 = 0,6383 / X_{1,1}^1 = 32,9303$$

$$X_{1,1}^2 = X_{1,1}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_1^1 - X_{1,1}^1|$$

$$X_{1,1}^2 = 32,9303 + 2,0 \cdot \cos(5,9577) \cdot |0,5062 \cdot 32,9303 - 32,9303|$$

$$X_{1,1}^2 = 63,7447$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^1 = 72,927 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,3557 = 2,2349 / r_3 = 2 \cdot 0,8085 = 1,617$$

$$r_4 = 0,3686 / X_{1,2}^1 = 72,927$$

$$X_{1,2}^2 = X_{1,2}^1 + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^1 - X_{1,2}^1|$$

$$X_{1,2}^2 = 72,927 + 2,0 \cdot \sin(2,2349) \cdot |1,617 \cdot 72,927 - 72,927|$$

$$X_{1,2}^2 = 143,7929$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^1 = 44,8614 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,8239 = 5,1767 / r_3 = 2 \cdot 0,0253 = 0,0506$$

$$r_4 = 0,6268 / X_{1,3}^1 = 44,8614$$

$$X_{1,3}^2 = X_{1,3}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_3^1 - X_{1,3}^1|$$

$$X_{1,3}^2 = 44,8614 + 2,0 \cdot \cos(5,1767) \cdot |0,0506 \cdot 44,8614 - 44,8614|$$

$$X_{1,3}^2 = 83,0069$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^1 = 32,9303 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,4507 = 2,8318 / r_3 = 2 \cdot 0,9661 = 1,9322$$

$$r_4 = 0,6625 / X_{2,1}^1 = -11,202$$

$$X_{2,1}^2 = X_{2,1}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_1^1 - X_{2,1}^1|$$

$$X_{2,1}^2 = -11,202 + 2,0 \cdot \cos(2,8318) \cdot |1,9322 \cdot 32,9303 + 11,202|$$

$$X_{2,1}^2 = -153,7376$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^1 = 72,927 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,7775 = 4,8852 / r_3 = 2 \cdot 0,7269 = 1,4538$$

$$r_4 = 0,767 / X_{2,2}^1 = 76,1347$$

$$X_{2,2}^2 = X_{2,2}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^1 - X_{2,2}^1|$$

$$X_{2,2}^2 = 76,1347 + 2,0 \cdot \cos(4,8852) \cdot |1,4538 \cdot 72,927 - 76,1347|$$

$$X_{2,2}^2 = 86,4128$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^1 = 44,8614 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,3926 = 2,4668 / r_3 = 2 \cdot 0,6348 = 1,2696$$

$$r_4 = 0,1056 / X_{2,3}^1 = 66,6762$$

$$X_{2,3}^2 = X_{2,3}^1 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^1 - X_{2,3}^1|$$

$$X_{2,3}^2 = 66,6762 + 2,0 \cdot \sin(2,4668) \cdot |1,2696 \cdot 44,8614 - 66,6762|$$

$$X_{2,3}^2 = 78,8213$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^1 = 32,9303 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,9273 = 5,8264 / r_3 = 2 \cdot 0,1494 = 0,2988$$

$$r_4 = 0,8934 / X_{3,1}^1 = 94,6737$$

$$X_{3,1}^2 = X_{3,1}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_1^1 - X_{3,1}^1|$$

$$X_{3,1}^2 = 94,6737 + 2,0 \cdot \cos(5,8264) \cdot |0,2988 \cdot 32,9303 - 94,6737|$$

$$X_{3,1}^2 = 246,9467$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^1 = 72,927 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,0276 = 0,1734 / r_3 = 2 \cdot 0,962 = 1,924$$

$$r_4 = 0,6456 / X_{3,2}^1 = -63,2213$$

$$X_{3,2}^2 = X_{3,2}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^1 - X_{3,2}^1|$$

$$X_{3,2}^2 = -63,2213 + 2,0 \cdot \cos(0,1734) \cdot |1,924 \cdot 72,927 + 63,2213|$$

$$X_{3,2}^2 = 337,74$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^1 = 44,8614 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,6371 = 4,003 / r_3 = 2 \cdot 0,6584 = 1,3168$$

$$r_4 = 0,493 / X_{3,3}^1 = 63,8176$$

$$X_{3,3}^2 = X_{3,3}^1 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^1 - X_{3,3}^1|$$

$$X_{3,3}^2 = 63,8176 + 2,0 \cdot \sin(4,003) \cdot |1,3168 \cdot 44,8614 - 63,8176|$$

$$X_{3,3}^2 = 56,6183$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^1 = 32,9303 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,092 = 0,5781 / r_3 = 2 \cdot 0,7013 = 1,4026$$

$$r_4 = 0,3982 / X_{4,1}^1 = 77,6001$$

$$X_{4,1}^2 = X_{4,1}^1 + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^1 - X_{4,1}^1|$$

$$X_{4,1}^2 = 77,6001 + 2,0 \cdot \sin(0,5781) \cdot |1,4026 \cdot 32,9303 - 77,6001|$$

$$X_{4,1}^2 = 111,9293$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^1 = 72,927 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,9946 = 6,2493 / r_3 = 2 \cdot 0,5665 = 1,133$$

$$r_4 = 0,6357 / X_{4,2}^1 = -94,4137$$

$$X_{4,2}^2 = X_{4,2}^1 + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^1 - X_{4,2}^1|$$

$$X_{4,2}^2 = -94,4137 + 2,0 \cdot \cos(6,2493) \cdot |1,133 \cdot 72,927 + 94,4137|$$

$$X_{4,2}^2 = 259,463$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^1 = 44,8614 / r_1 = 2,0$$

$$r_2 = 2 \cdot \pi \cdot 0,7924 = 4,9788 / r_3 = 2 \cdot 0,254 = 0,508$$

$$r_4 = 0,1595 / X_{4,3}^1 = -11,8695$$

$$X_{4,3}^2 = X_{4,3}^1 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^1 - X_{4,3}^1|$$

$$X_{4,3}^2 = -11,8695 + 2,0 \cdot \sin(4,9788) \cdot |0,508 \cdot 44,8614 - -11,8695|$$

$$X_{4,3}^2 = -78,7423$$

SCA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 1:

- ind 1: [63.7447, **143.7929**, 83.0069], **una dimensión infactible**
- ind 2: [-**153.7376**, 86.4128, 78.8213], **una dimensión infactible**
- ind 3: [**246.9467**, **337.74**, 56.6183], **dos dimensiones infactibles**
- ind 4: [**111.9293**, **259.463**, -78.7423]. **dos dimensions infactibles**

Reparación de soluciones:

- ind 1: [63.7447, 100.0, 83.0069] / fitness: 20953.5322
- ind 2: [-100.0, 86.4128, 78.8213] / fitness: 23679.9693
- ind 3: [100.0, 100.0, 56.6183] / fitness: 23205.6319
- ind 4: [100.0, 100.0, -78.7423] / fitness: 26200.3498

Mejor solución:

- ind 1 (inicial): [32.9303, 72.927, 44.8614] / fitness: 8415.2972

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^2 = 32,9303 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,6075 = 3,817 / r_3 = 2 \cdot 0,836 = 1,672$$

$$r_4 = 0,3958 / X_{1,1}^2 = 63,7447$$

$$X_{1,1}^3 = X_{1,1}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^2 - X_{1,1}^2|$$

$$X_{1,1}^3 = 63,7447 + 1,98 \cdot \sin(3,817) \cdot |1,672 \cdot 32,9303 - 63,7447|$$

$$X_{1,1}^3 = 52,993$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^2 = 72,927 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,3791 = 2,382 / r_3 = 2 \cdot 0,2897 = 0,5794$$

$$r_4 = 0,0516 / X_{1,2}^2 = 100,0$$

$$X_{1,2}^3 = X_{1,2}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^2 - X_{1,2}^2|$$

$$X_{1,2}^3 = 100,0 + 1,98 \cdot \sin(2,382) \cdot |0,5794 \cdot 72,927 - 100,0|$$

$$X_{1,2}^3 = 178,7356$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^2 = 44,8614 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,1929 = 1,212 / r_3 = 2 \cdot 0,1893 = 0,3786$$

$$r_4 = 0,1087 / X_{1,3}^2 = 83,0069$$

$$X_{1,3}^3 = X_{1,3}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^2 - X_{1,3}^2|$$

$$X_{1,3}^3 = 83,0069 + 1,98 \cdot \sin(1,212) \cdot |0,3786 \cdot 44,8614 - 83,0069|$$

$$X_{1,3}^3 = 205,4067$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^2 = 32,9303 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,6396 = 4,0187 / r_3 = 2 \cdot 0,6756 = 1,3512$$

$$r_4 = 0,4889 / X_{2,1}^2 = -100,0$$

$$X_{2,1}^3 = X_{2,1}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^2 - X_{2,1}^2|$$

$$X_{2,1}^3 = -100,0 + 1,98 \cdot \sin(4,0187) \cdot |1,3512 \cdot 32,9303 + 100,0|$$

$$X_{2,1}^3 = -319,9809$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^2 = 72,927 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,4158 = 2,6125 / r_3 = 2 \cdot 0,9603 = 1,9206$$

$$r_4 = 0,5317 / X_{2,2}^2 = 86,4128$$

$$X_{2,2}^3 = X_{2,2}^2 + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^2 - X_{2,2}^2|$$

$$X_{2,2}^3 = 86,4128 + 1,98 \cdot \cos(2,6125) \cdot |1,9206 \cdot 72,927 - 86,4128|$$

$$X_{2,2}^3 = -5,2907$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^2 = 44,8614 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,2638 = 1,6575 / r_3 = 2 \cdot 0,3344 = 0,6688$$

$$r_4 = 0,249 / X_{2,3}^2 = 78,8213$$

$$X_{2,3}^3 = X_{2,3}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^2 - X_{2,3}^2|$$

$$X_{2,3}^3 = 78,8213 + 1,98 \cdot \sin(1,6575) \cdot |0,6688 \cdot 44,8614 - 78,8213|$$

$$X_{2,3}^3 = 175,1178$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^2 = 32,9303 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,4432 = 2,7847 / r_3 = 2 \cdot 0,57 = 1,14$$

$$r_4 = 0,3918 / X_{3,1}^2 = 100,0$$

$$X_{3,1}^3 = X_{3,1}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^2 - X_{3,1}^2|$$

$$X_{3,1}^3 = 100,0 + 1,98 \cdot \sin(2,7847) \cdot |1,14 \cdot 32,9303 - 100,0|$$

$$X_{3,1}^3 = 143,2058$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^2 = 72,927 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,9695 = 6,0915 / r_3 = 2 \cdot 0,7389 = 1,4778$$

$$r_4 = 0,5002 / X_{3,2}^2 = 100,0$$

$$X_{3,2}^3 = X_{3,2}^2 + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^2 - X_{3,2}^2|$$

$$X_{3,2}^3 = 100,0 + 1,98 \cdot \cos(6,0915) \cdot |1,4778 \cdot 72,927 - 100,0|$$

$$X_{3,2}^3 = 115,1058$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^2 = 44,8614 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,8239 = 5,1767 / r_3 = 2 \cdot 0,349 = 0,698$$

$$r_4 = 0,4263 / X_{3,3}^2 = 56,6183$$

$$X_{3,3}^3 = X_{3,3}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^2 - X_{3,3}^2|$$

$$X_{3,3}^3 = 56,6183 + 1,98 \cdot \sin(5,1767) \cdot |0,698 \cdot 44,8614 - 56,6183|$$

$$X_{3,3}^3 = 11,8188$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^2 = 32,9303 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,2275 = 1,4294 / r_3 = 2 \cdot 0,9632 = 1,9264$$

$$r_4 = 0,1771 / X_{4,1}^2 = 100,0$$

$$X_{4,1}^3 = X_{4,1}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^2 - X_{4,1}^2|$$

$$X_{4,1}^3 = 100,0 + 1,98 \cdot \sin(1,4294) \cdot |1,9264 \cdot 32,9303 - 100,0|$$

$$X_{4,1}^3 = 171,6724$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^2 = 72,927 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,6642 = 4,1733 / r_3 = 2 \cdot 0,7949 = 1,5898$$

$$r_4 = 0,2639 / X_{4,2}^2 = 100,0$$

$$X_{4,2}^3 = X_{4,2}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^2 - X_{4,2}^2|$$

$$X_{4,2}^3 = 100,0 + 1,98 \cdot \sin(4,1733) \cdot |1,5898 \cdot 72,927 - 100,0|$$

$$X_{4,2}^3 = 72,916$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^2 = 44,8614 / r_1 = 1,98$$

$$r_2 = 2 \cdot \pi \cdot 0,2943 = 1,8491 / r_3 = 2 \cdot 0,0782 = 0,1564$$

$$r_4 = 0,1825 / X_{4,3}^2 = -78,7423$$

$$X_{4,3}^3 = X_{4,3}^2 + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^2 - X_{4,3}^2|$$

$$X_{4,3}^3 = -78,7423 + 1,98 \cdot \sin(1,8491) \cdot |0,1564 \cdot 44,8614 - -78,7423|$$

$$X_{4,3}^3 = 84,5263$$

SCA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 2:

- ind 1: [52.993, **178.7356**, **205.4067**], **dos dimensiones infactibles**
- ind 2: [**-319.9809**, -5.2907, **175.1178**], **dos dimensiones infactibles**
- ind 3: [**143.2058**, **115.1058**, 11.8188], **dos dimensiones infactibles**
- ind 4: [**171.6724**, 72.916, 84.5263], **una dimensión infactible**

Reparación de soluciones:

- ind 1: [52.993, 100.0, 100.0] / fitness: 22808.258
- ind 2: [-100.0, -5.2907, 100.0] / fitness: 20027.9915
- ind 3: [100.0, 100.0, 11.8188] / fitness: 20139.684
- ind 4: [100.0, 72.916, 84.5263] / fitness: 22461.4384

Mejor solución:

- ind 1 (inicial): [32.9303, 72.927, 44.8614] / fitness: 8415.2972

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^{100} = -0,006 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,5842 = 3,6706 / r_3 = 2 \cdot 0,7469 = 1,4938$$

$$r_4 = 0,3017 / X_{1,1}^{100} = -3,1921$$

$$X_{1,1}^{101} = X_{1,1}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^{100} - X_{1,1}^{100}|$$

$$X_{1,1}^{101} = -3,1921 + 0,02 \cdot \sin(3,6706) \cdot |1,4938 \cdot -0,006 + 3,1921|$$

$$X_{1,1}^{101} = -3,2242$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^{100} = -0,027 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,1929 = 1,212 / r_3 = 2 \cdot 0,3093 = 0,6186$$

$$r_4 = 0,3236 / X_{1,2}^{100} = -0,443$$

$$X_{1,2}^{101} = X_{1,2}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^{100} - X_{1,2}^{100}|$$

$$X_{1,2}^{101} = -0,443 + 0,02 \cdot \sin(1,212) \cdot |0,6186 \cdot -0,027 + 0,443|$$

$$X_{1,2}^{101} = -0,435$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^{100} = 0,0166 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,3534 = 2,2205 / r_3 = 2 \cdot 0,4831 = 0,9662$$

$$r_4 = 0,7587 / X_{1,3}^{100} = -0,021$$

$$X_{1,3}^{101} = X_{1,3}^{100} + r_1 \cdot \cos(r_2) \cdot |r_3 P_3^{100} - X_{1,3}^{100}|$$

$$X_{1,3}^{101} = -0,021 + 0,02 \cdot \cos(2,2205) \cdot |0,9662 \cdot 0,0166 + 0,021|$$

$$X_{1,3}^{101} = -0,0214$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^{100} = -0,006 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,7749 = 4,8688 / r_3 = 2 \cdot 0,5361 = 1,0722$$

$$r_4 = 0,0658 / X_{2,1}^{100} = 0,0011$$

$$X_{2,1}^{101} = X_{2,1}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^{100} - X_{2,1}^{100}|$$

$$X_{2,1}^{101} = 0,0011 + 0,02 \cdot \sin(4,8688) \cdot |1,0722 \cdot -0,006 - 0,0011|$$

$$X_{2,1}^{101} = 0,001$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^{100} = -0,027 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,3604 = 2,2645 / r_3 = 2 \cdot 0,2793 = 0,5586$$

$$r_4 = 0,3739 / X_{2,2}^{100} = -0,0688$$

$$X_{2,2}^{101} = X_{2,2}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^{100} - X_{2,2}^{100}|$$

$$X_{2,2}^{101} = -0,0688 + 0,02 \cdot \sin(2,2645) \cdot |0,5586 \cdot -0,027 + 0,0688|$$

$$X_{2,2}^{101} = -0,068$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^{100} = 0,0166 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,7147 = 4,4906 / r_3 = 2 \cdot 0,3972 = 0,7944$$

$$r_4 = 0,0488 / X_{2,3}^{100} = 0,0215$$

$$X_{2,3}^{101} = X_{2,3}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^{100} - X_{2,3}^{100}|$$

$$X_{2,3}^{101} = 0,0215 + 0,02 \cdot \sin(4,4906) \cdot |0,7944 \cdot 0,0166 - 0,0215|$$

$$X_{2,3}^{101} = 0,0213$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^{100} = -0,006 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,9148 = 5,7479 / r_3 = 2 \cdot 0,7066 = 1,4132$$

$$r_4 = 0,3865 / X_{3,1}^{100} = 0,0015$$

$$X_{3,1}^{101} = X_{3,1}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_1^{100} - X_{3,1}^{100}|$$

$$X_{3,1}^{101} = 0,0015 + 0,02 \cdot \sin(5,7479) \cdot |1,4132 \cdot -0,006 - 0,0015|$$

$$X_{3,1}^{101} = 0,0014$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^{100} = -0,027 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,4266 = 2,6804 / r_3 = 2 \cdot 0,7012 = 1,4024$$

$$r_4 = 0,6116 / X_{3,2}^{100} = -0,0543$$

$$X_{3,2}^{101} = X_{3,2}^{100} + r_1 \cdot \cos(r_2) \cdot |r_3 P_2^{100} - X_{3,2}^{100}|$$

$$X_{3,2}^{101} = -0,0543 + 0,02 \cdot \cos(2,6804) \cdot |1,4024 \cdot -0,027 + 0,0543|$$

$$X_{3,2}^{101} = -0,0546$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^{100} = 0,0166 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,5109 = 3,2101 / r_3 = 2 \cdot 0,5778 = 1,1556$$

$$r_4 = 0,1903 / X_{3,3}^{100} = -0,1612$$

$$X_{3,3}^{101} = X_{3,3}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^{100} - X_{3,3}^{100}|$$

$$X_{3,3}^{101} = -0,1612 + 0,02 \cdot \sin(3,2101) \cdot |1,1556 \cdot 0,0166 + 0,1612|$$

$$X_{3,3}^{101} = -0,1614$$

Ecuación general SCA:

$$X_{ij}^{t+1} = \begin{cases} X_{ij}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{ij}^t| & , r_4 < 0,5 \\ X_{ij}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{ij}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_1^{100} = -0,006 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,6409 = 4,0269 / r_3 = 2 \cdot 0,9187 = 1,8374$$

$$r_4 = 0,6185 / X_{4,1}^{100} = 0,0259$$

$$X_{4,1}^{101} = X_{4,1}^{100} + r_1 \cdot \cos(r_2) \cdot |r_3 P_1^{100} - X_{4,1}^{100}|$$

$$X_{4,1}^{101} = 0,0259 + 0,02 \cdot \cos(4,0269) \cdot |1,8374 \cdot -0,006 - 0,0259|$$

$$X_{4,1}^{101} = 0,0254$$

Ecuación general SCA:

$$X_{ij}^{t+1} = \begin{cases} X_{ij}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{ij}^t| & , r_4 < 0,5 \\ X_{ij}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{ij}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_2^{100} = -0,027 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,5654 = 3,5525 / r_3 = 2 \cdot 0,923 = 1,846$$

$$r_4 = 0,4064 / X_{4,2}^{100} = -0,0398$$

$$X_{4,2}^{101} = X_{4,2}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_2^{100} - X_{4,2}^{100}|$$

$$X_{4,2}^{101} = -0,0398 + 0,02 \cdot \sin(3,5525) \cdot |1,846 \cdot -0,027 + 0,0398|$$

$$X_{4,2}^{101} = -0,0399$$

Ecuación general SCA:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^t + r_1 \cdot \sin(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 < 0,5 \\ X_{i,j}^t + r_1 \cdot \cos(r_2) \cdot |r_3 P_j^t - X_{i,j}^t| & , r_4 \geq 0,5 \end{cases}$$

$$P_3^{100} = 0,0166 / r_1 = 0,02$$

$$r_2 = 2 \cdot \pi \cdot 0,7751 = 4,8701 / r_3 = 2 \cdot 0,4663 = 0,9326$$

$$r_4 = 0,4804 / X_{4,3}^{100} = 0,0145$$

$$X_{4,3}^{101} = X_{4,3}^{100} + r_1 \cdot \sin(r_2) \cdot |r_3 P_3^{100} - X_{4,3}^{100}|$$

$$X_{4,3}^{101} = 0,0145 + 0,02 \cdot \sin(4,8701) \cdot |0,9326 \cdot 0,0166 - 0,0145|$$

$$X_{4,3}^{101} = 0,0145$$

SCA: Ejemplo práctico - validación restricciones

Restricción: $x_1, x_2, x_3 \in [-100, 100]$

Soluciones obtenidas en la iteración 100:

ind 1: [-3.2242, -0.435, -0.0214], solución factible

ind 2: [0.001, -0.068, 0.0213], solución factible

ind 3: [0.0014, -0.0546, -0.1614], solución factible

ind 4: [0.0254, -0.0399, 0.0145], solución factible

Reparación de soluciones:

ind 1: [-3.2242, -0.435, -0.0214] / fitness: 10.5851

ind 2: [0.001, -0.068, 0.0213] / fitness: 0.0051

ind 3: [0.0014, -0.0546, -0.1614] / fitness: 0.029

ind 4: [0.0254, -0.0399, 0.0145] / fitness: 0.0024

Mejor solución:

ind 4: [0.0254, -0.0399, 0.0145] / fitness: 0.0024

SCA: Ejemplo práctico - resultado final

Soluciones obtenidas:

ind 1: [-3.2242, -0.435, -0.0214] / fitness: 10.5851

ind 2: [0.001, -0.068, 0.0213] / fitness: 0.0051

ind 3: [0.0014, -0.0546, -0.1614] / fitness: 0.029

ind 4: [0.0254, -0.0399, 0.0145] / fitness: 0.0024

Mejor solución:

ind 4: [0.0254, -0.0399, 0.0145] / fitness: 0.0024